Tanta University

Electrical Power and Machines Engineering
Electromagnetic Fields: Sheet 1

Faculty of Engineering

1. Given the vectors $\overline{\mathbf{M}} = -10\overline{\mathbf{a}}_\mathbf{x} + 4\overline{\mathbf{a}}_\mathbf{y} - 8\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{N}} = 8\overline{\mathbf{a}}_\mathbf{x} + 7\overline{\mathbf{a}}_\mathbf{y} - 2\overline{\mathbf{a}}_\mathbf{z}$, find (a)a unit vector in the direction of $-\overline{\mathbf{M}} + 2\overline{\mathbf{N}}$; (b) the magnitude of $5\overline{\mathbf{a}_\mathbf{x}} + \overline{\mathbf{N}} - 3\overline{\mathbf{M}}$ and (c) the value of $|\overline{\mathbf{M}}||2\overline{\mathbf{N}}|(\overline{\mathbf{M}} + \overline{\mathbf{N}})$.

2. Vector $\overline{\mathbf{A}}$ extends from the origin to point $(1, 2, 3)$, and vector $\overline{\mathbf{B}}$ extends from the origin to (2, 3, -2). Find (a) the unit vector in the direction of $(\overline{\mathbf{A}} - \overline{\mathbf{B}})$; (b)the unit vector in the direction of the line extending from the origin to the midpoint of the line joining the ends of $\overline{\mathbf{A}}$ and $\overline{\mathbf{B}}$.

3. The vector from the origin to point A is given as (6, -2, -4), and the unit vector directed from the origin toward point B is $(2, -2, 1)/3$. If points A and B are ten units apart, find the coordinates of point B.

4. Find the acute angle between the two vectors $\overline{\mathbf{A}} = 2\overline{\mathbf{a}}_\mathbf{x} + \overline{\mathbf{a}}_\mathbf{y} + 3\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{B}} = \overline{\mathbf{a}}_\mathbf{x} - 3\overline{\mathbf{a}}_\mathbf{y} + 2\overline{\mathbf{a}}_\mathbf{z}$ by using the definition of (a)the dot product and (b) the cross product.

5. Given the points $M(0.1, -0.2, -0.1)$, $N(-0.2, 0.1, 0.3)$, and $P(0.4, 0, 0.1)$, find (a) the vector $\overline{\mathbf{R}}_{MN}$; (b) the dot product $\overline{\mathbf{R}}_{MN}.\overline{\mathbf{R}}_{MP}$; (c) the scalar projection of $\overline{\mathbf{R}}_{MN}$ on $\overline{\mathbf{R}}_{MP}$; (d) the angle between $\overline{\mathbf{R}}_{MN}$ and $\overline{\mathbf{R}}_{MP}$.

6. Find (a) the vector component of $\overline{\mathbf{F}} = 10\overline{\mathbf{a}}_\mathbf{x} - 6\overline{\mathbf{a}}_\mathbf{y} + 5\overline{\mathbf{a}}_\mathbf{z}$ that is parallel to $\overline{\mathbf{G}} = 0.1\overline{\mathbf{a}}_\mathbf{x} + 0.2\overline{\mathbf{a}}_\mathbf{y}0.3\overline{\mathbf{a}}_\mathbf{z}$; (b) the vector component of $\overline{\mathbf{F}}$ that is perpendicular to $\overline{\mathbf{G}}$; (c) the vector component of $\overline{\mathbf{G}}$ that is perpendicular to $\overline{\mathbf{F}}$.

7. Three vectors extending from the origin are given as $\overline{\mathbf{R}}_1 = (7, 3, -2)$, $\overline{\mathbf{R}}_2 = (-2, 7, -3)$, and $\overline{\mathbf{r}}_3 = (0, 2, 3)$. Find (a) a unit vector perpendicular to both $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$; (b) a unit vector perpendicular to the vectors $\overline{\mathbf{R}}_1$ - $\overline{\mathbf{R}}_2$ and $\overline{\mathbf{R}}_2$ - $\overline{\mathbf{R}}_3$; (c) the area of the triangle defined by $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$; (d) the area of the triangle defined by the heads of $\overline{\mathbf{R}}_1$, $\overline{\mathbf{R}}_2$, and $\overline{\mathbf{R}}_3$.

8. Express in cylindrical components: (a) the vector from $C(3, 2, -7)$ to $D(-1, -4, 2)$; (b) a unit vector at D directed toward C; (c) a unit vector at D directed toward the origin.

9. The surfaces $r = 2$ and 4, $\theta = 30^\circ$ and $50\circ$, and $\phi = 20^\circ$ and 60° identify a closed surface. Find (a) the enclosed volume; (b) the total area of the enclosing surface; (c) the total length of the twelve edges of the surface; (d) the length of the longest straight line that lies entirely within the surface.

10. Express the unit vector ax in spherical components at the point: (a) $r = 2\text{m}$, $\theta = 1\text{rad}$, $\phi = 0.8\text{rad}$; (b) $x = 3\text{m}$, $y = 2\text{m}$, $z = -1\text{m}$; (c)$\rho = 2.5\text{m}$, $\phi = 0.7\text{rad}$, $z = 1.5\text{m}$.

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1. Given the vectors M = −10ax +4ay −8az and N = 8ax +7ay −2az, FIND :
(a)a unit vector in the direction of −M + 2N;
(b) the magnitude of 5ax + N − 3M
(c) the value of |M||2N|(M + N).

a)

$$-M + 2N = -(-10\text{ax} + 4\text{ay} - 8\text{az}) + 2(8\text{ax} + 7\text{ay} - 2\text{az})$$
$$= \left(10\, a_x - 4a_y + 8\, a_z\right) + \left(16\, a_x + 14\, a_y - 4\, a_z\right) = \mathbf{26\, a_x + 10\, a_y + 4\, a_z}$$

$$|-M + 2N| = \sqrt{26^2 + 10^2 + 4^2} = 6\sqrt{22}$$

$$unit\ vector = \frac{26\, a_x + 10\, a_y + 4\, a_z}{6\sqrt{22}} = 0.9238\, a_x + 0.355334\, a_y + 0.142133\, a_z$$

b)

$$5a_x + \text{N} - 3\text{M} = 5\, \text{a}_\text{x} + \left(8\, a_x + 7\, a_y + 2\, a_z\right) - 3\left(-10\, a_x + 4\, a_y - 8\, a_z\right)$$

$$= 43\, a_x - 5\, a_y - 6\, a_z$$
$$|5a_x + \text{N} - 3\text{M}| = \sqrt{43^2 + 5^2 + 6^2} = 43.7035$$

c)

$$|M| = \sqrt{10^2 + 4^2 + 8^2} = 6\sqrt{5} = 13.4164$$
$$|2N| = \sqrt{16^2 + 14^2 - 4^2} = 2\sqrt{109} = 20.88061$$
$$\bar{M} + \bar{N} = \left(-10\, a_x + 4\, a_y - 8\, a_z\right) + \left(8\, a_x + 7\, a_y + 2\, a_z\right) = -2\, a_x + 11\, a_y - 6\, a_z$$
$$\therefore \quad |M||2N|(\bar{M} + \bar{N}) = (13.416)(20.88)\left(-2\, a_x + 11\, a_y - 6\, a_z\right) = -560.25\, a_x + 3081.38\, a_y - 1680.75\, a_z$$

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2. Vector A extends from the origin to point (1, 2, 3), and vector B extends from the origin to (2, 3, -2). Find
(a) the unit vector in the direction of (A−B);
(b)the unit vector in the direction of the line extending from the origin to the midpoint of the line joining the ends of A and B.

a)

$$\bar{A} = 1\, a_x + 2\, a_y + 3\, a_z$$
$$\bar{B} = 2\, a_x + 3\, a_y - 2\, a_z$$

$$\bar{A} - \bar{B} = \left(1\, a_x + 2\, a_y + 3\, a_z\right) - \left(2\, a_x + 3\, a_y - 2\, a_z\right)$$
$$= -1\, a_x - 1\, a_y + 1\, a_z$$
$$|\bar{A} - \bar{B}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$unit\ vector = \frac{-1\, a_x - 1\, a_y + 1a_z}{\sqrt{3}} = -\frac{1}{\sqrt{3}}\, a_x - \frac{1}{\sqrt{3}}\, a_y + \frac{1}{\sqrt{3}}\, a_z$$

b)

$$the\ midpoint\ = \frac{1+2}{2}\ a_x\ + \frac{2+3}{2}\ a_y\ + \frac{3+2}{2}\ a_z = 1.5\ a_x + 2.5\ a_y + 2.5\ a_z$$

$$it's\ magnitude\ = \sqrt{1.5^2 + 2.5^2 + 2.5^2}\ = 3.34057$$

$$unit\ vector\ = \frac{1.5\ a_x + 2.5\ a_y + 2.5\ a_z}{3.34057} = 0.3905\ a_x\ + 0.6509\ a_y + 0.6509\ a_z$$

3. The vector from the origin to point A is given as (6, -2, -4), and the unit vector directed from the origin toward point B is (2, −2, 1)/3. If points A and B are ten units apart, find the coordinates of point B.

$$A = 6\ a_x - 2\ a_y\ - 4\ a_z$$
$$B\ unit\ vector\ = \frac{2}{3}\ a_x\ - \frac{2}{3}\ a_y\ + \frac{1}{3}\ a_z$$
$$\therefore\ \ B = \frac{2}{3} k\ a_x\ - \frac{2}{3}\ k\ a_y\ + \frac{1}{3}\ k\ a_z$$

$$\because\ \ |A\bar{B}| =\ |B - A| = 10\ units$$
$$\sqrt{\left(\frac{2}{3}k - 6\right)^2 + \left(-\frac{2}{3}k + 2\right)^2 + \left(\frac{1}{3}k + 4\right)^2} = 10$$
$$\left(\frac{2}{3}k - 6\right)^2 + \left(-\frac{2}{3}k + 2\right)^2 + \left(\frac{1}{3}k + 4\right)^2 = 100$$
$$put\ \ C = \frac{2}{3}k$$

$$(\ C - 6)^2 + (-C + 2)^2\ + \left(\frac{C}{2} + 4\right)^2\ = 100$$
$$C^2 - 12C + 36\ \ + C^2 - 4C + 4\ \ \ \ + \frac{C^2}{4} + 4C + 16 = 100$$
$$\left(1 + 1 + \frac{1}{4}\right)C^2 + (\ -12 - 4 + 4)C + (\ 36 + 4 + 16\ ) = 100$$
$$2.25C^2 - 12C - 44 = 0$$
$$\therefore C\ =\ 7.8306\ \ \ \ \ or\ \ \ - 2.497$$

$$\therefore k = \frac{3}{2}\ C =\ \ 11.7459\ \ \ \ or\ \ - 3.7455$$
$$\therefore\ \ B = \frac{2}{3}k\ a_x\ - \frac{2}{3}\ k\ a_y\ + \frac{1}{3}\ k\ a_z$$
$$\boldsymbol{\bar{B} = 7.8306\ a_x - 7.8306\ a_y\ \ + 3.9153\ a_z}$$

$$or$$

$$\boldsymbol{\bar{B}\ \ = -2.497\ a_x\ \ + 2.497\ a_y\ \ - 1.2458\ a_z}$$

4. Find the acute angle between the two vectors A = 2ax + ay + 3az and B = ax − 3ay + 2az by using the definition of (a)the dot product and (b) the cross product.

a) using the dot product

$$A = \sqrt{2^2 + 1^2 + 3^2}\ \ \ = \sqrt{14}\ = 3.741$$
$$B = \sqrt{1^2 + 3^2 + 2^2}\ \ \ \ = \sqrt{14}\ = 3.741$$
$$\bar{A} \cdot \bar{B}\ = (2a_x\ +\ a_y\ +\ 3a_z\ ) \cdot (a_x\ -\ 3a_y\ +\ 2a_z\ ))$$

$$= 2 - 3 + 6 = 5 = ABcos(\theta) = 14\cos(\theta)$$

$$\therefore \cos(\theta) = \frac{5}{14}$$

$$\therefore \theta = \cos^{-1}\left(\frac{5}{14}\right) = \ 69.07\,° \ = 1.2\,rad$$

b) using the cross product

$$\bar{A} \times \bar{B} \ = \begin{bmatrix} a_x & a_y & a_z \\ 2 & 1 & 3 \\ 1 & -3 & 2 \end{bmatrix} = (2+9)a_x \ -(4-3)a_y \ +(-6-1)a_z \ = 11a_x - a_y - 7a_z$$

$$\therefore |\bar{A} \times \bar{B}\ | = \sqrt{11^2 + 1^2 + 7^2} \ = 3\sqrt{19} = 13.0766 = AB\sin(\theta) = 14\sin(\theta)$$

$$\therefore \sin(\theta) = \frac{3\sqrt{19}}{14}$$

$$\therefore \theta = \sin^{-1}\left(\frac{3\sqrt{19}}{14}\right) \ 69.07\,° \ = 1.2\,rad$$

5.Given the points $M\,(0.1, -0.2, -0.1)$, $N\,(-0.2, 0.1, 0.3)$, and $P\,(0.4, 0, 0.1)$, find

(a)the vector R_{MN} ; (b) the dot product $R_{MN} . R_{MP}$; (c) the scalar projection of R_{MN} on R_{MP} ; (d) the angle between R_{MN} and R_{MP} .

a)

$$\overline{R_{MN}} \ = \bar{N} - \bar{M} = \ -0.3\ a_x \ \ + 0.3\ a_y + 0.4\ a_z$$

$$|\overline{R_{MN}}| = \sqrt{0.3^2 + \ -0.3^2 + 0.4^2} = 0.583$$

$$\overline{R_{MP}} \ = \bar{P} - \bar{M} = 0.3\ a_x \ + 0.2\ a_y \ + 0.2\ a_z$$

$$|\overline{R_{MP}}| \ = \sqrt{0.3^2 + 0.2^2 + 0.2^2} = 0.41231$$

b)

$$\overline{R_{MN}} \cdot \overline{R_{MP}} \ \ = \left(-0.3\ a_x \ \ + 0.3\ a_y + 0.4\ a_z\right) \cdot \left(0.3\ a_x \ + 0.2\ a_y \ + 0.2\ a_z\right)$$

$$= \ -0.09 + 0.06 + 0.08 = 0.05$$

c)

$$the\ scalar\ projection \ = \frac{R_{MN} \ \cdot \ R_{MP}}{|\overline{R_{MP}}|} = \frac{0.05}{0.41231} = 0.12126$$

d)

$$\overline{R_{MN}} \ \cdot \overline{R_{MP}} \ \ = |\overline{R_{MN}}||\overline{R_{MP}}|\cos(\theta) = 0.583 * 0.4123\cos(\theta) = 0.05$$

$$\therefore \theta \ = \cos^{-1}(\frac{0.05}{0.240}) = 77.9\,° \approx \ \ 78\,° \ \ = 1.361\,rad$$

6. Find (a) the vector component of F = 10ax − 6ay + 5az that is parallel to G = 0.1ax + 0.2ay+0.3az; (b) the vector component of F that is perpendicular to G; (c) the vector component of G that is perpendicular to F.

$$|F| = \sqrt{10^2 + 6^2 + 5^2} = 12.688$$
$$|G| = \sqrt{0.1^2 + 0.2^2 + 0.3^2} = 0.37416$$

a)

$$= \frac{F \cdot G}{|\bar{G}|} = \frac{(1 + (-1.2) + 1.5)}{0.37416} = 3.474$$
$$\overline{a_g} = \frac{0.1\mathrm{a_x} + 0.2\mathrm{a_y} + 0.3\mathrm{a_z}}{0.37416} = 0.2672\, a_x + 0.5345\, a_y + 0.8018\, a_z$$
$$\therefore\ F\ parallel\ component\ to\ G = 3.474\,(0.2672\, a_x + 0.5345\, a_y + 0.8018\, a_z)$$
$$\overline{\boldsymbol{F}_{\parallel}} = \mathbf{0.928}\, \boldsymbol{a_x} + \mathbf{1.85}\, \boldsymbol{a_y} + \mathbf{2.78}\, \boldsymbol{a_z}$$

b)

$$F\ perpendicular\ to\ G$$

$$\bar{F} = \overline{F_{\parallel}} + \overline{F_{\perp}}$$
$$\therefore\ \overline{F_{\perp}} = \bar{F} - \overline{F_{\parallel}} = (10\mathrm{a_x} - 6\mathrm{a_y} + 5\mathrm{a_z}) - (0.928\, a_x + 1.85\, a_y + 2.78\, a_z)$$
$$\overline{\boldsymbol{F}_{\perp}} = \mathbf{9.072}\, \boldsymbol{a_x} - \mathbf{7.85}\, \boldsymbol{a_y} + \mathbf{2.22}\, \boldsymbol{a_z}$$

c)

$$G\ \ component\ perpendicular\ on\ F$$

$$\overline{G_{\perp}} = \bar{G} - \overline{G_{\parallel}}$$
$$= (0.1a_x + 0.2a_y + 0.3a_z) - \left(\frac{F \cdot G}{|\bar{F}|}\right)\overline{a_f}$$
$$(0.1a_x + 0.2a_y + 0.3a_z) - \left(\frac{(1 + (-1.2) + 1.5)}{12.688}\right)\overline{a_f}$$
$$\overline{a_f} = \frac{(10a_x - 6a_y + 5a_z)}{12.688} = 0.788\, a_x - 0.472\, a_y + 0.39\, a_z$$
$$\therefore\ \left(\frac{F \cdot G}{|\bar{F}|}\right)\overline{a_f} = 0.102\,(0.788\, a_x - 0.472\, a_y + 0.39\, a_z) = 0.080376\, a_x - 0.048144\, a_y + 0.03978\, a_z$$
$$\therefore\ \overline{G_{\perp}} = (0.1a_x + 0.2a_y + 0.3a_z) - (0.080376\, a_x - 0.048144\, a_y + 0.03978\, a_z)$$
$$\overline{\boldsymbol{G}_{\perp}} = \mathbf{0.019624}\, \boldsymbol{a_x} + \mathbf{0.248144}\, \boldsymbol{a_y} + \mathbf{0.26022}\, \boldsymbol{a_z}$$

7. Three vectors extending from the origin are given as R1 = (7, 3, −2), R2 = (−2, 7, −3), and R3 = (0, 2, 3). Find (a) a unit vector perpendicular to both R1 and R2; (b) a unit vector perpendicular to the vectors R1 - R2 and R2 - R3; (c) the area of the triangle defined by R1 and R2; (d) the area of the triangle defined by the heads of R1, R2, and R3.

a)

$$a\ vector\ \bar{n}\ is\ perpendicular\ on\ \ R\bar{1}\ \ and\ \overline{R_2}\ \ if$$
$$\bar{n} = \overline{R_1} \times \overline{R_2} = (7,3,-2) \times (-2,7,-3) = \begin{bmatrix} a_x & a_y & a_z \\ 7 & 3 & -2 \\ -2 & 7 & -3 \end{bmatrix} = 5\, a_x + 25\, a_y + 55a_z$$

$$\therefore \ a \ unit \ vector \ \overline{a_n} = \frac{5\,a_x + 25\,a_y + 55a_z}{\sqrt{5^2+25^2+55^2}} = \left(\frac{\sqrt{(75626\,)} * \mathrm{a_x} + \sqrt{(75626\,)} * 5\mathrm{a_y} + \sqrt{(75626\,)} * 11\mathrm{a_z}}{75626}\right) =$$

$$\boldsymbol{\overline{a_n} = 0.00363\,a_x + 0.018181\,a_y + 0.0399\,a_z}$$

b)

$$\overline{R_1} - \overline{R_2} = (7,3,-2) - (-2\,,7,-3) = 9\,a_x - 4\,a_y + a_z$$

$$\overline{\mathrm{R_2}} - \overline{\mathrm{R_3}} = (-2\,,7,-3) - (0,2,3) = -2\mathrm{a_x} + 5\mathrm{a_y} - 6\,\mathrm{a_z}$$

$a \ vector \ \ z \ will \ be \ perpendicular \ on \ both \ of \ them \ if$

$$z = (\overline{R_1} - \overline{R_2}) \times \ (\overline{\mathrm{R_2}} - \overline{\mathrm{R_3}}\ ) = \begin{bmatrix} a_x & a_y & a_z \\ 9 & -4 & 1 \\ -2 & 5 & -6 \end{bmatrix} = 19\,a_x + 52\,a_y + 37\,a_z$$

$$\therefore \ \overline{a_z} \ = \frac{19\,a_x + 52\,a_y + 37\,a_z}{\sqrt{19^2+52^2+37^2}} = \frac{19\,a_x + 52\,a_y + 37\,a_z}{66.588287258346} = \boldsymbol{0.285\,a_x + 0.7809\,a_y + 0.556\,a_z}$$

c)

$$the \ area \ of \ the \ triangle = \frac{1}{2}|\,\overline{R_1} \times \overline{R_2}\,|$$

$$\overline{R_1} \times \overline{R_2} = = 5\,a_x + 25\,a_y + 55a_z \ from \ (a)$$

$$\therefore \ \frac{1}{2}|\,\overline{R_1} \times \overline{R_2}\,| = \frac{1}{2} * \sqrt{5^2+25^2+55^2} \ = \ \boldsymbol{30.311 \ Square \ Units}$$

d)

$$the \ two \ vectors \ that \ determine \ this \ triangle \ are \ \overline{R_1R_2} \ \ and \ \overline{R_1R_3}$$

$$\overline{R_1R_2} = \overline{R_2} - \overline{R_1} \ = \ -9\,a_x \ + 4\,a_y - 1\,a_z$$

$$\overline{R_1R_3} = \overline{R_3} - \overline{R_1} \ = \ -7\,a_x - 1\,a_y + 5\,a_z$$

$$\therefore \ the \ Area \ = \ \frac{1}{2} \ |\,\overline{R_1R_2} \ \times \ \overline{R_1R_3}\,|$$

$$\overline{R_1R_2} \ \times \ \overline{R_1R_3} \ = \begin{bmatrix} a_x & a_y & a_z \\ -9 & 4 & -1 \\ -7 & -1 & 5 \end{bmatrix} = 19\,a_x + 52\,a_y + 37\,a_z$$

$$\therefore \ = \frac{1}{2} \ |\,\overline{R_1R_2} \ \times \ \overline{R_1R_3}\,| = \frac{1}{2} \ \sqrt{19^2+52^2+37^2} \ \ = \boldsymbol{33.294 \ \ square \ units}$$

8.Express in cylindrical components: (a) the vector from $C(3, 2, -7)$ to $D(-1, -4, 2)$;(b) a unit vector at D directed toward C; (c) a unit vector at D directed toward the origin.

a)

$$The \ vector \ \ \overline{CD} \ \ = \overline{D} - \bar{C} \ = \ -4\,a_x - 6\,a_y + 9\,a_z$$

to $to\ cylindrical \quad \overline{CD} * \overline{a_\rho} \quad = -4\, a_x \cdot \overline{a_\rho} - 6\, a_y \cdot \overline{a_\rho} + 9\, a_z \cdot \overline{a_\rho} = -4\cos(\phi) - 6\sin(\phi)$

$$\overline{CD} * \overline{a_\phi} \quad = -4\, a_x \cdot \overline{a_\phi} - 6\, a_y \cdot \overline{a_\phi} + 9\, a_z \cdot \overline{a_\phi} = 4\sin(\phi) \ - 6\cos(\phi)$$

$$\overline{CD} * \overline{a_z} \quad = -4\, a_x \cdot \overline{a_z} - 6\, a_y \cdot \overline{a_z} + 9\, a_z \cdot \overline{a_z} = 9$$

$$\therefore in\ cylinderical\ \overline{CD} \ = (-4\cos(\phi) - 6\sin(\phi))\overline{a_\rho} \quad + (4\sin(\phi) \ - 6\cos(\phi))\overline{a_\phi} + 9\, a_z$$

$$where\ \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad , for\ point\ (x, y, z)$$

b) a unit vector $\overline{DC}$

$$\overline{DC} \quad = \bar{C} - \bar{D} = (3,2,-7) \ - (-1, -4, 2) = 4\, a_x + 6\, a_y - 9\, a_z$$

$$\therefore \quad unit\ vector \ = \frac{4\, a_x + 6\, a_y - 9\, a_z}{\sqrt{4^2 + 6^2 + 9^2}} = 0.347\, a_x + 0.52\, a_y - 0.78\, a_z$$

$$\widehat{DC}(unit\ vector) \ * \overline{a_\rho} \quad = 0.347\ \cos(\phi) + 0.347\sin(\phi)$$

$$\widehat{DC}(unit\ vector) \ * \overline{a_\phi} = -\ 0.347\ \sin(\phi) + 0.347\cos(\phi)$$

$$\widehat{DC}(unit\ vector) \ * \overline{a_z} = -0.78$$

$$\therefore \ in\ cylendrical\ \widehat{DC} = \ (0.347\ \cos(\phi) + 0.347\sin(\phi))a_\rho + (-\ 0.347\ \sin(\phi) + 0.347\cos(\phi))a_\phi - 0.78\, a_z$$

$$where\ \ \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$C)\ \overline{DO} \ = (0,0,0) - (-1, -4, 2) = 1\, a_x + 4\, a_y - 2\, a_z$

$$\widehat{DO} \ = \frac{1\, a_x + 4\, a_y - 2\, a_z}{\sqrt{1^2 + 4^2 + 2^2}} = 0.218\, a_x + 0.873\, a_y - 0.436\, a_z$$

$to\ cylendrical:$

$$(0.218\cos(\phi) + 0.873\sin(\phi))a_\rho \ + \ (-0.218\sin\phi + 0.873\cos\phi)a_\phi - 0.436\, a_z$$

9- The surfaces r = 2 and 4, θ = 30◦ and 50◦, and φ = 20◦ and 60◦ identify a closed surface. Find (a) the enclosed volume; (b) the total area of the enclosing surface; (c) the total length of the twelve edges of the surface; (d) the length of the longest straight line that lies entirely within the surface.

10- Express the unit vector ax in spherical components at the point:
(a) r = 2m, θ = 1rad, φ = 0.8rad; (b) x = 3m, y = 2m, z = −1m; (c)ρ = 2.5m, φ = 0.7rad, z = 1.5m.

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$

$$a_x = (sin\theta\cos\phi)\overline{a_r} + (\cos\theta\cos\phi)\overline{a_\theta} - \sin\phi\ a_\phi$$

a)

$$a_x = (sin1\cos 0.8)\overline{a_r} + (\cos 1\cos 0.8)\overline{a_\theta} - \sin 0.8\ a_\phi$$
$$= (0.5863)\overline{a_r} + (0.3764)\overline{a_\theta} - 0.7174\ a_\phi$$

b)

$$first\ we\ convert\ the\ point\ to\ spherical\ cordinates$$
$$r = \sqrt{x^2+y^2+z^2} = \sqrt{3^2+2^2+-1^2} = \sqrt{14} = 3.7417$$
$$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{-1}{\sqrt{14}}\right) = 105.5014°$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{3}\right) = 33.6901°$$

$$\therefore\ a_x = (sin105.5\cos 33.69)\overline{a_r} + (\cos 105.5\cos 33.69)\overline{a_\theta} - \sin 33.69\ a_\phi$$
$$= (0.8018)\overline{a_r} + (-0.2224)\overline{a_\theta} - 0.5547\ a_\phi$$

c)

$$convert\ it\ to\ cartezian$$
$$x = \rho\cos\phi = 2.5\cos 0.7 = 1.9121\ m$$
$$y = \rho\sin\phi = 2.5\sin 0.7 = 1.6105\ m$$
$$z = 1.5\ m$$
$$to\ spherical$$
$$r = \sqrt{1.9^2+1.6^2+1.5^2} = 2.9017$$
$$\theta = \cos^{-1}\left(\frac{1.5}{2.9}\right) = 58.8526\ °$$
$$\phi = \tan^{-1}\left(\frac{1.6}{1.9}\right) = 40.1009\ °$$

$$a_x = (sin58.8526\ \cos 40.1)\overline{a_r} + (\cos 58.8526\ \cos 40.1)\overline{a_\theta} - \sin 40.1\ a_\phi$$
$$= (0.6546)\overline{a_r} + (0.3956)\overline{a_\theta} - 0.6441\ a_\phi$$

1. **Find** the volume defined by:
(a) $3 < r < 5$, $0.1\pi < \theta < 0.3\pi$ and $1.2\pi < \phi < 1.6\pi$
(b) $4 < \rho < 6$, $2 < Z < 5$ and $30^\circ < \phi < 60^\circ$

2. **Find** the area of:
(a) Curved surface of right circular cylinder with $\rho = 2$ m, $0 < Z < 5$ and $30^\circ < \phi < 120^\circ$
(b) Strip $0 < \theta < \pi$ on a spherical shell of radius r

3. **Transfer** $\mathbf{G} = \frac{XZ}{Y}\,\mathbf{a_x}$ into:
(a) Cylindrical coordinates system (b) Spherical coordinates system

4. Using the coordinate system named, **give** vector at point A (2,1,-3) that extends to B (1,3,4) using: (a) Cartesian (b) Cylindrical (c) Spherical

5. **Find** the gradient of theses scalar fields:
(a) $U = 4xz^2 + 3yz$
(b) $W = 2\rho\,(z^2 + 1)\cos\phi$
(c) $H = r^2 \cos\theta \cos\emptyset$

6. In the region of free space that includes the volume, $2 < x, y, z < 3$,
$$\mathbf{D} = \frac{2}{z^2}\left(yz\,\mathbf{a_x} + xz\,\mathbf{a_y} - 2xz\,\mathbf{a_z}\right)$$
Evaluate the divergence of **D.**

7. If $\mathbf{E} = \frac{16}{r} \cos 2\theta\ \mathbf{a}_{\boldsymbol{\theta}}$.**Evaluate** the divergence of **E.**

8. Let $\mathbf{D} = \left(2\rho\, z^2\ \mathbf{a_\rho} + \rho\cos^2\phi\ \mathbf{a_z}\right)$. **Evaluate**
(a) $\oint \mathbf{D}.\,d\mathbf{S}$
(b) $\int \nabla.\mathbf{D}\,dv$
over the region defined by $0 \le \rho \le 5,\ 0 < \phi < 2\pi,\ -1 \le z \le 1.$

Sheet ① " Vectors Analysis "

Ex ①: given: vectors

$\bar{M} = -10\,\bar{a}_x + 4\bar{a}_y - 8\,\bar{a}_z$

$\bar{N} = 8\,\bar{a}_x + 7\bar{a}_y - 2\bar{a}_z$

Find: a) unit vector in $(-\bar{M}+2\bar{N})$ direction.

b) $|5\bar{a}_x + \bar{N} - 3\bar{M}|$

c) $|M|\,|2N|\,(\bar{M}+\bar{N})$

Solution

a) let: $\bar{A} = -\bar{M}+2\bar{N}$

$= 10\,\bar{a}_x - 4\bar{a}_y + 8\,\bar{a}_z + 16\,\bar{a}_x + 14\,\bar{a}_y - 4\bar{a}_z$

$= 26\,\bar{a}_x + 10\,\bar{a}_y + 4\,\bar{a}_z$

$|\bar{A}| = \sqrt{(26)^2+(10)^2+(4)^2} = 28.142$

$\therefore\ \bar{a}_A = \dfrac{26\,\bar{a}_x + 10\,\bar{a}_y + 4\bar{a}_z}{28.142} = \dfrac{\bar{A}}{|A|}$

$= 0.923\,\bar{a}_x + 0.355\,\bar{a}_y + 0.142\,\bar{a}_z$

b) let: $\bar{B} = 5\,\bar{a}_x + \bar{N} - 3\bar{M}$

$= 5\bar{a}_x + 8\bar{a}_x + 7\bar{a}_y - 2\,\bar{a}_z$

$+ 30\,\bar{a}_x - 12\,\bar{a}_y + 24\,\bar{a}_z$

$= 43\,\bar{a}_x - 5\bar{a}_y + 22\,\bar{a}_z$

$\therefore\ |\bar{B}| = \sqrt{(43)^2 + (-5)^2 + (22)^2}$

c) let $\bar{C} = |M|\,|2N|\,(\bar{M}+\bar{N})$

c) $|\bar{M}| = \sqrt{100+16+64} = 13.416$

$|\bar{N}| = \sqrt{64+49+4} = 10.8167$

$(\bar{M}+\bar{N}) = 2\bar{a}_x + 11\bar{a}_y - 10\bar{a}_z$

$\therefore C = 13.416 * 10.8167 \; (2\bar{a}_x + 11\bar{a}_y - 10\bar{a}_z)$

$= -580.5\,\bar{a}_x + 3193\,\bar{a}_y - 2902\,\bar{a}_z$

$= (-580.5, 3193, 2902)$

anotter form to represet vector

Ex ②: given: Vectors

$\bar{OA}: (0,0,0) \rightarrow (1,2,3) \Rightarrow a_x + 2\bar{a}_y + 3\bar{a}_z$

$\bar{OB}: (0,0,0) \rightarrow (2,3,-2) \Rightarrow 2a_x + 3\bar{a}_y - 2\bar{a}_z$

Find: a) $\bar{a}_{(\bar{A}-\bar{B})}$

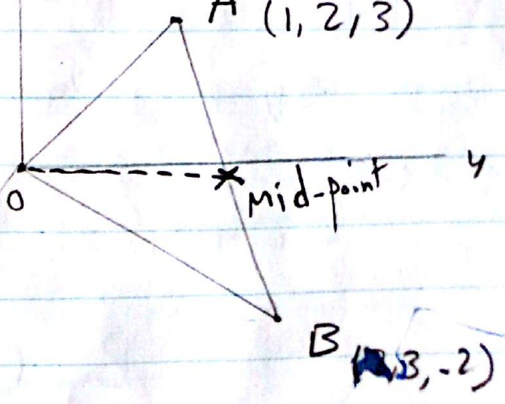
b) Unit vector in the direction of line extending from (0,0,0) to the mid Point of the line joining the ends of A and B.

Solution

a) $\bar{A}-\bar{B} = -\bar{a}_x - \bar{a}_y + 5\bar{a}_z$

$|A-B| = \sqrt{1+1+25} = 5.196$

$\therefore \bar{a}_{(\bar{A}-\bar{B})} = \dfrac{-a_x - 4a_y + 5a_z}{5.196}$



$M = \frac{1}{2}(A+B) \leftarrow$ mid-point احداثيات

$\frac{1}{2}(3,5,1)$

$\therefore \bar{M} = (3\bar{a}_x + 5\bar{a}_y + \bar{a}_z)\frac{1}{2}$

$\therefore |\bar{M}| = \sqrt{\quad} = 2.958$

$\therefore \bar{a}_{OM} = \frac{\bar{M}}{|\bar{M}|}$

$= 0.571\,\bar{a}_x + 0.84527\,\bar{a}_y + 0.169\,\bar{a}_z$

Ex ③: given: $\bar{A} = (6,-2,-4)$, $\bar{a}_{AB} = (2,-2,1)/3$

$|\bar{AB}| = 10$ Find B ? (B_x, B_y, B_z)

Solution

$\bar{AB} = |\bar{AB}|\,\bar{a}_{AB} = B - A$

$\frac{10}{3}(2,-2,1) = (B_x - 6)\bar{a}_x + (B_y + 2)\bar{a}_y + (B_z + 4)\bar{a}_z$

$\therefore B_x - 6 = \frac{20}{3} \rightarrow B_x = 12.66$

$B_y + 2 = \frac{20}{3} \rightarrow B_y = -8.66$

$B_z + 4 = \frac{10}{3} \rightarrow B_z = -0.66$

$\therefore B\,(12.66, -8.66, -0.66)$

Ex $\boxed{4}$: given: Two vectors $\bar{A} = 2\bar{a}_x + \bar{a}_y + 3\bar{a}_z$
$\bar{B} = \bar{a}_x - 3\bar{a}_y + 2\bar{a}_y$. Find: The acute angle using · dot product · cross product

solution

· $\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}| \cos\theta = 2 + (-3) + 6$

$\sqrt{1+9+4} * \sqrt{4+1+9} \cdot \cos\theta = 5$

$3.741 * 3.741 \cos\theta = 5$

$\theta = \cos^{-1}(\quad) = 69.07°$

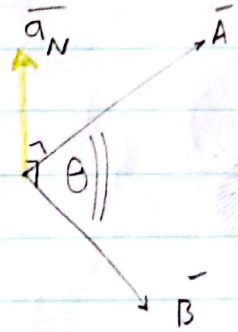
· $\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta \, \bar{a}_N = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 2 & 1 & 3 \\ 1 & -3 & 2 \end{vmatrix}$

$3.741 * 3.741 \sin\theta \, \cancel{\bar{a}_N}$

$= 11\bar{a}_x - 4\bar{a}_y - 7\bar{a}_z$

$= |\bar{A} \times \bar{B}| \, \cancel{a}_N$

$= \sqrt{11^2 + 4^2 + 7^2}$



$\therefore \theta = \sin^{-1}(\quad) = 69.07°$

Ex $\boxed{5}$: given: points $M(0.1, -0.2, -0.1)$
$N(-0.2, 0.1, 0.3)$, $P(0.4, 0, 0.1)$

Find: a) $\bar{R}_{MN}$ b) $\bar{R}_{MN} \cdot \bar{R}_{MP}$
c) Scalar projection of $\bar{R}_{MN}$ on $\bar{R}_{MP}$
d) angle betw R_{MN} & R_{MP}

Solution

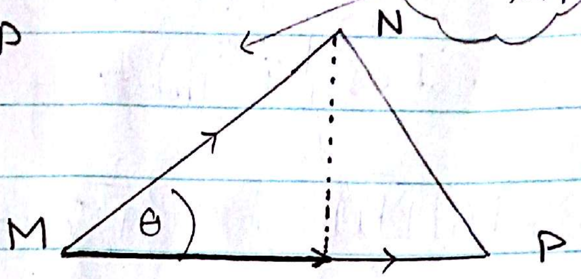
$\rightarrow$

a) $\bar{R}_{MN} = N - M = (-0.3,\ 0.3,\ 0.4)$

$\bar{R}_{MP} = P - M = (0.3,\ 0.2,\ 0.2)$

b) $\bar{R}_{MN} \cdot \bar{R}_{MP} = -0.09 + 0.06 + 0.08 = 0.05$

c) scalar projection of $\bar{R}_{MN}$ on R_{MP} $= \bar{R}_{MN} \cdot \bar{a}_{R_{MP}}$



$= |\bar{R}_{MN}| \cos\theta$

$= \dfrac{\bar{R}_{MN} \cdot \bar{R}_{MP}}{|\bar{R}_{MP}|}$

$= \dfrac{0.05}{0.412} = 0.1213$

d) $0.583 \cos\theta = 0.1213$

$\therefore \theta = \cos^{-1}(\quad) = 78°$

Ex $\boxed{6}$: given: $\bar{F} = 10\,\bar{a}_x - 6\,\bar{a}_y + 5\,\bar{a}_y$

a) Find vector Component of $\bar{F}$ that is Parallel to $G = 0.1\,\bar{a}_x + 0.2\,\bar{a}_y + 0.3\,\bar{a}_z$.

b) " " " F Pendicular " " G.

c) " " " G " " " F.

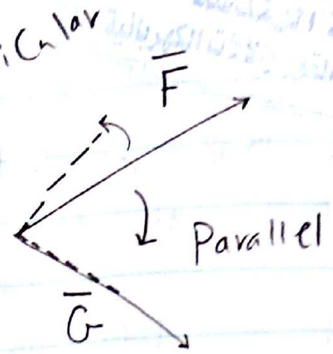
solution

$\bar{F}_{\parallel G}$

$= (\bar{F} \cdot \bar{a}_G)\, \bar{a}_G$

قيمة مسقط متجه F على $\bar{G}$

اتجاه مركبة الموازية لمتجه G



$\bar{a}_G = \dfrac{0.1\bar{a}_x + 0.2a_y + 0.3\bar{a}_z}{0.5916 = \sqrt{0.1^2 + 0.2^2 + 0.3^2}}$

$= (0.93, 1.86, 2.79)$

$\bar{F}_{\perp G}$: $\because \bar{F} =$ مركبة موازية + مركبة عمودية

$F_{\parallel} + \bar{F}_{\perp}$

$\therefore F_{\perp G} = \bar{F} - \bar{F}_{\parallel G}$

$= (9.07, -7.86, 2.21)$

Ex (7): given, Three vectors

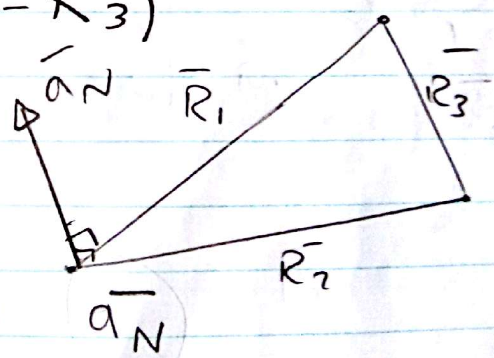
$\bar{R}_1 = (7, 3, -2)$, $R_2 = (-2, 7, 3)$

$\bar{R}_3 = (0, 2, 3)$

a) $\bar{a}_N$ $(\bar{R}_1 - \bar{R}_2)$ and $(R_2 - \bar{R}_3)$

b) $\bar{a}_N$ $\bar{R}_1$ and $\bar{R}_2$

c) area of the triangle defined by $\bar{R}_1, \bar{R}_2$



a) $\bar{R}_1 \times \bar{R}_2$

$= |R_1||R_2| \sin\theta$?

$= |R_1 \times R_2|\, \bar{a}_N$

$\therefore \bar{a}_N = \dfrac{R_1 \times R_2}{|R_1 \times R_2|} = \dfrac{5, 25, 55}{60.6} = (0.08, 0.41, 0.91)$

b) $\bar{R}_1 - \bar{R}_2 = (9,-4,1) = \bar{A}$

$\bar{R}_2 - \bar{R}_3 = (-2,5,-6) = \bar{B}$

$\therefore \bar{A}\times\bar{B} = |\bar{A}\times\bar{B}|\ \bar{a}_{N_{AB}}$

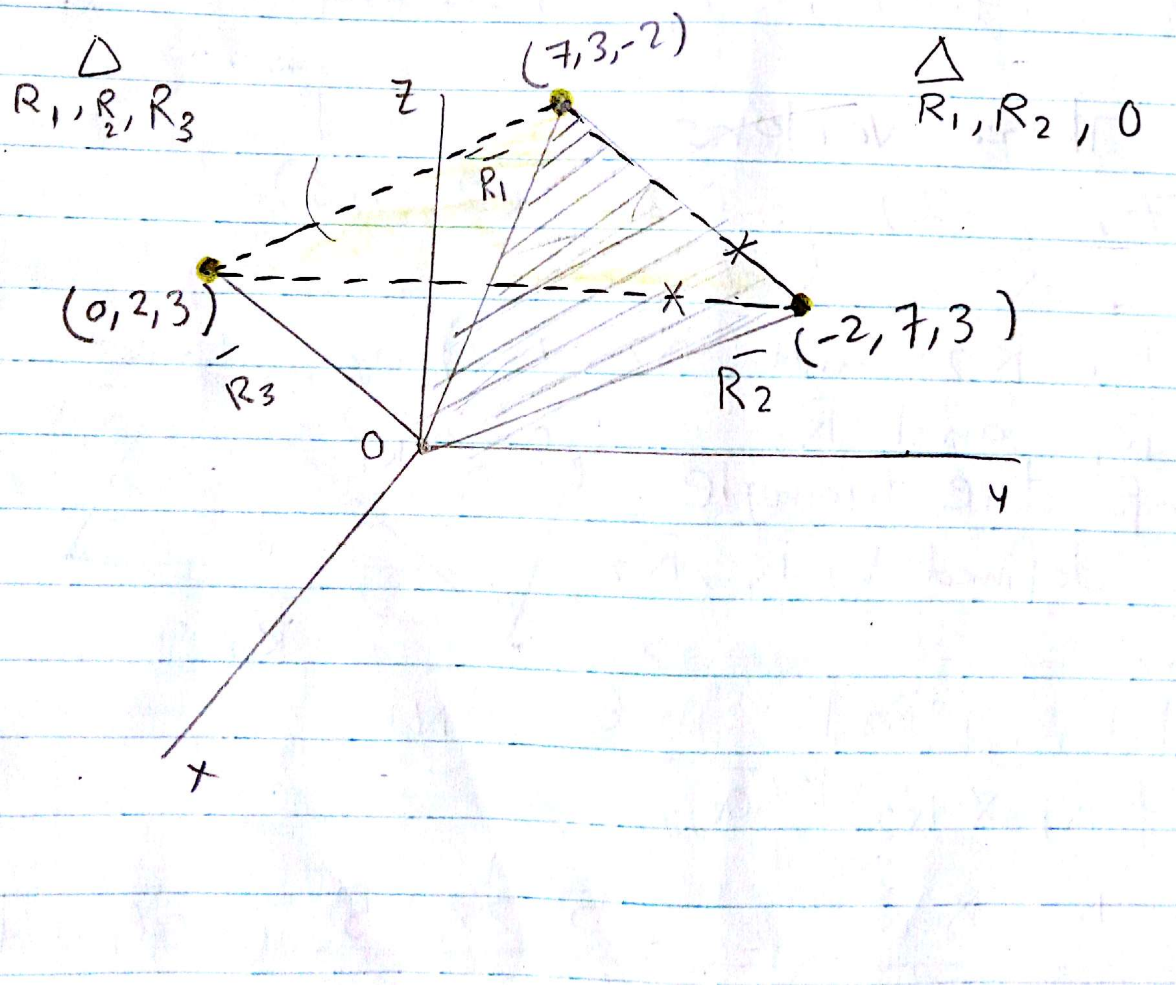
$$\bar{a}_{N_{AB}} = \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|} = \frac{(19,92,32)}{63.95} = (0.3,0.81,0.5)$$

c) Area of $\underset{\bar{R}_1,\bar{R}_2}{\Delta} = \frac{1}{2}|\bar{R}_1\times\bar{R}_2|$

$= 30.3$

d) Area of $\underset{\bar{R}_1,\bar{R}_2,\bar{R}_3}{\Delta} = \frac{1}{2}|(R_1-R_2)\times(R_3-R_2)|$

$= 32$



Ex [9] given: The surfaces

$r = 2, 4$, $\theta = 30^\circ, 50^\circ$, $\phi = 20^\circ, 60^\circ$

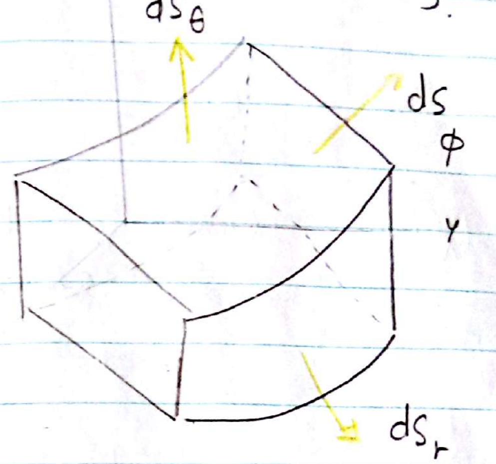
identify a closed surface. Find:

(a) Volume (b) A_{tot} (c) ℓ (d) $\ell_{largest}$

Solution

spherical coordinates

dr, $r\,d\theta$, $r\sin\theta\,d\phi$



(a) $dv = r^2 \sin\theta\,dr\,d\theta\,d\phi$

$$\therefore V = \int dv$$

$$= \int_{\phi=20}^{60^\circ} \int_{\theta=30}^{50^\circ} \int_{r=2}^{4} dv$$

$$= \frac{r^3}{3}\Big|_2^4 * -\cos\theta\Big|_{30^\circ}^{50^\circ} * \phi\Big|_{20*\frac{\pi}{180}}^{60*\frac{\pi}{180}}$$

$$= 2.91 \rightarrow \text{check}$$

(b) $A = \int ds$

$$= \int_{\phi=20}^{60} \int_{\theta=30}^{50} r^2 \Big|_{r=2,4} \sin\theta\,d\theta\,d\phi + \int_{\phi=20}^{60} \int_{r=2}^{4} r\sin\theta\,dr \Big|_{\theta=30,}$$

$$+ \int_{\theta=30}^{50} \int_{r=2}^{4} r\,dr\,d\theta \Big|_{\phi=20,\,60} = 12.61$$

Scanned by CamScanner

Ex [10] given: Point

(a) $r = 2$, $\theta = 1$ rad, $\phi = 0.8$ rad

(b) $x = 3$, $y = 2$, $z = -1$

(c) $\rho = 2.5$, $\phi = 0.7$ rad, $z = 1.5$

Express the unit vector a_x in spherical Compents

Solution

From Table:

Unit vector a_x

Point

$$\bar{a}_x = \sin\theta \cos\phi \, \bar{a}_r + \cos\theta \cos\phi \, \bar{a}_\theta + -\sin\phi \, \bar{a}_\phi \rightarrow (*)$$

a) at $(2, 1 \text{ rad}, 0.8 \text{ rad}) \rightarrow \therefore \bar{a}_x = $ ✓

b) Point (x, y, z) $\longrightarrow$ Point (r, θ, ϕ)

$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{14}$

$\theta = \cos^{-1} \frac{z}{r} = 105.5°$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = 33.7°$

$\therefore \bar{a}_x = $ ✓

c) Point (ρ, ϕ, z) $\longrightarrow$ Point (r, θ, ϕ)

$r = \sqrt{\rho^2 + z^2} = \sqrt{8.5}$

$\theta = \cos^{-1} \frac{z}{r} = 59°$

$\phi = 0.7$ rad

$\therefore \bar{a}_x = $ ✓

sheet ②   " Vector Algebra and CalCuls "

Ex $\boxed{1}$: Find the volume defined by:

(a) $3 < r < 5$, $0.1\pi < \theta < 0.3\pi$, $1.2\pi < \phi < 1.6\pi$

(b) $4 < \rho < 6$, $2 < Z < 5$, $30^\circ < \phi < 60^\circ$

Solution

(a) Using spherical Coordinates (r, θ, ϕ)

$$dv = r^3 \sin\theta \, dr \, d\theta \, d\phi$$

$$\therefore V = \int_v dv = \int_{\phi=1.2\pi}^{1.6\pi} \int_{\theta=0.1\pi}^{0.3\pi} \int_{r=3}^{5} dv$$

$$= \frac{r^3}{3}\Big|_3^5 * -\cos\theta \Big|_{0.1\pi \times \frac{180}{\pi}}^{0.3\pi \times \frac{180}{\pi}} \phi \Big|_{1.2\pi}^{1.6\pi} = 14.91 \text{ unit}^3$$

(b) Using Cylindrical (ρ, ϕ, Z)

$$dv = \rho \, d\rho . \, d\phi \, dZ$$

$$V = \int_v dv = \int_{Z=2}^{5} \int_{\phi=30^\circ}^{60} \int_{\rho=4}^{6} dv$$

$$= \frac{\rho^2}{2}\Big|_4^6 * \phi \Big|_{\frac{30 \times \pi}{180}}^{\frac{60 \times \pi}{180}} * Z \Big|_2^5 = 5\pi$$

قطاع أشريط

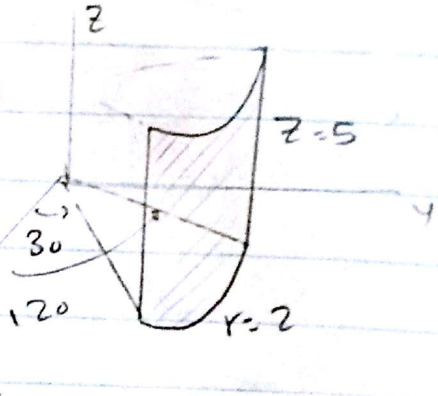
Ex [2]: Find the area of
(a) surface of $\rho = 2m$, $0 < z < 5$, $30° < \phi < 120°$.
(b) strip $0 < \theta < \pi$ on a spherical shell of reduis r.

يا
قشرة كروية
$0 < \phi < 2\pi$

Solution

(a) $\because$ ρ Const

$$\bar{S}_{\rho} = \int_{s} d\bar{S}_{\rho} = \int_{z=0}^{5} \int_{\phi=30}^{120} \rho \, d\phi \, dz$$



$$\therefore \bar{S} = 5\pi \, m^2 \, \bar{a}_{\rho}$$

(b) $\because$ r Const

$$\therefore \bar{S}_r = \int_s d\bar{S}_r = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$\bar{S}_r = 4\pi r^2 \bar{a}_r \rightarrow$$ مساحة كرة كاملة
وهذا متوقع

Ex [4] give vector $\overline{AB}$

$A(2,1,-3)$ $B(1,3,4)$

using (a) Cartesian

(b) cylindrical

(c) spherical

solution

(a) Cartesian

$\overline{AB} = B - A$

$= -\overline{a}_x + 2\overline{a}_y + 7\overline{a}_z$

(b) cylindrical

$AB = AB_\rho\, \overline{a}_\rho + AB_\phi\, \overline{a}_\phi + AB_z\, \overline{a}_z$

$AB_\rho = \overline{AB} \cdot \overline{a}_\rho = (-\overline{a}_x + 2\overline{a}_y + 7\overline{a}_z) \cdot \overline{a}_\rho$

From Table

$= -\cos\phi + 2\sin\phi + 0$

$= \text{zero}$

$AB_\phi = \overline{AB} \cdot \overline{a}_\phi$

$= \sin\phi + 2\cos\phi + 0$

$= 2.236$

$AB_z = 7$

AT Point A

$\phi = \tan^{-1}\frac{y}{x}$

$= 26.56$

$\rho = \sqrt{x^2+y^2}$

$= \sqrt{5}$

$\therefore \overline{R}_{AB} = 2.236\,\overline{a}_\phi + 7\,\overline{a}_z$

(c) spherical

From Table

$AB_r = \overline{AB} \cdot \overline{a}_r = -5.61$

$AB_\theta = \overline{AB} \cdot \overline{a}_\theta = -4.18$

$AB_\phi = AB \cdot a_\phi = 2.236$

AT A

$\phi = \tan^{-1}\frac{y}{x}$

$= 26.56$

$\theta = \cos^{-1}\frac{z}{r}$

$= 143.3$

$\therefore \overline{AB} = -5.61\,\overline{a}_r - 4.18\,\overline{a}_\theta + 2.236\,\overline{a}_\phi$

Ex $\boxed{5}$: Find the gradient of these scalar fields:

(a) $U = 4xz^2 + 3yz$

(b) $W = 2\rho(z^2+1)\cos\phi$ $= 2\rho z^2\cos\phi + 2\rho\cos\phi$

(c) $H = r^2\cos\theta\cos\phi$

مؤثر تفاضلي (نبلا)

Solution

gradient $\Rightarrow \nabla(\ \ ) = \boxed{\ }$ متجه

كمية قياسية

(a) ∇U (Cartesian)

$$\nabla U = \frac{\partial U}{\partial x}\bar{a}_x + \frac{\partial U}{\partial y}\bar{a}_y + \frac{\partial U}{\partial z}\bar{a}_z$$

$$= 4z^2\,\bar{a}_x + 3z\,\bar{a}_y + (8xz + 3y)\,\bar{a}_z$$

(b) ∇W (Cylindrical)

$$\nabla W = \frac{\partial W}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial W}{\partial \phi}\bar{a}_\phi + \frac{\partial W}{\partial z}\bar{a}_z$$

$$= 2(z^2+1)\cos\phi\,\bar{a}_\rho + \frac{-1}{\cancel{\rho}}\,2\cancel{\rho}\,(z^2+1)\,\bar{a}_\rho$$

$$+ 4\rho\cos\phi\,\bar{a}_z$$

(c) ∇H (Spherical)

$$\nabla H = \frac{\partial H}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial H}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial H}{\partial \phi}\bar{a}_\phi$$

$$= 2r\cos\theta\cos\phi\,\bar{a}_r + \frac{1}{\cancel{r}}\,r^{\cancel{2}}(-\sin\theta)\cos\phi\,\bar{a}_\theta$$

$$+ \frac{1}{r\sin\theta}\,r^2\cos\theta\,(-\sin\phi)\,\bar{a}_\phi$$

* Div. Theory :

$\rho_v = \nabla \cdot \bar{D}$ $\quad \oint_s D \cdot ds = \int \rho_v \, dv = \int_v (\nabla \cdot D) \, dv$

Ex $\boxed{5}$: given $2 < x, y, z < 3$

$$\bar{D} = \frac{2}{z^2}(yz\,\bar{a}_x + xz\,\bar{a}_y - 2xz\,\bar{a}_z)$$

Evaluate divergence of $\bar{D}$. ~~for the volume defined by~~ $2 < x, y, z < 3$

Solution

Divergence $\bar{\nabla} \cdot$ (متجه) $= \boxed{}$ كمية قياسية

$$\bar{\nabla} \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

(Cartesian)

$$\bar{D} = \underbrace{\frac{2y}{z}}_{D_x}\bar{a}_x + \underbrace{\frac{2x}{z}}_{D_y}\bar{a}_y + \underbrace{\frac{-4x}{z}}_{D_z}\bar{a}_z$$

$D_z = -4xz^{-1}$

$$\therefore \bar{\nabla} \cdot \bar{D} = \text{Zero} + \text{Zero} + 4xz^{-2}$$

$$= \frac{4x}{z^2}$$

Ex $\boxed{8}$: given:

$$\bar{E} = 15\rho^2 \sin 2\phi \ \bar{a}_\rho + 20\rho^2 \cos 2\phi \ \bar{a}_\phi$$

Evaluat $\bar{\nabla}\cdot\bar{E}$? for the region

$1 < \rho < 2m$, $1 < \phi < 2$ rad , $1 < z < 2m$

solution.

$$\underset{\text{(cylindrical)}}{\bar{\nabla}\cdot\bar{E}} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho E_\rho) + \frac{1}{r}\frac{\partial E_\phi}{\partial\phi} + \frac{\partial E_z}{\partial z}$$

مستقلة ر   ثابتة ، مشتقة

Ex $\boxed{7}$ given $\bar{E} = \frac{16}{r}\cos 2\theta \ \bar{a}_\theta$

Evaluate $\bar{\nabla}\cdot\bar{E}$ ~~for $1<r<2, 1<\phi<2, 1<\theta<2$~~

solution

$$\underset{\text{(spherical)}}{\bar{\nabla}\cdot\bar{E}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \underbrace{\cancel{E_r}}_{0}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \ E_\theta)$$

$$+ \frac{1}{r\sin\theta}\underbrace{\cancel{\frac{\partial E_\phi}{\partial\phi}}}_{0}$$

ثابت

$$= \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \ \frac{16}{r}\cos 2\theta\right)$$

$$= \frac{16}{r^2\cancel{\sin\theta}}\left\{\cancel{\sin\theta}(-2\sin 2\theta) + \cos 2\theta \ \cos\theta\right\}$$

$$= \frac{16}{r^2}\left\{-2\sin 2\theta + \cos 2\theta \ \frac{\cos\theta}{\sin\theta}\right\}$$

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Ex $\boxed{8}$ given: $\bar{D} = 2\rho Z^2 \bar{a}_\rho + \rho \cos^2\phi \, \bar{a}_z$

Evaluate: (a) $\oint_S \bar{D} \cdot \bar{ds}$

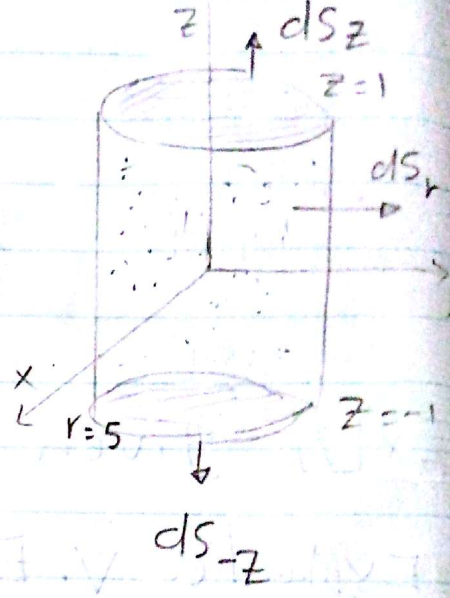
(b) $\int_V \nabla \cdot D \, dv$

over the region defined by:

$\le \rho \le 5, \; 0 < \phi < 2\pi, \; -1 \le Z \le 1 \Rightarrow$ اسطوانة

Solution

$d\rho, \; \rho d\phi, \; dZ$



(a) $\oint_S \bar{D} \cdot \bar{ds}$

$$= \underset{\substack{Z=-1 \\ (ds_{-z})}}{\iint} + \underset{\substack{Z=1 \\ (ds_z)}}{\iint} + \underset{\substack{\rho=5 \\ (ds_\rho)}}{\iint}$$

$$= \cancel{-\int_{\phi=0}^{2\pi} \int_{\rho=0}^{5} (\rho \cos^2\phi)(\rho \, d\rho \, d\phi)} \quad (Z=-1)$$

$$+ \cancel{\int_{\phi=0}^{2\pi} \int_{\rho=0}^{5} (\rho \cos^2\phi)(\rho \, d\rho \, d\phi)} \quad (Z=1)$$

$$+ \int_{Z=-1}^{1} \int_{\phi=0}^{2\pi} (2\rho Z^2)(\rho \, d\phi \, dz) \quad (\rho=5)$$

$$= \frac{200\pi}{3}$$

(b) $\bar{\nabla} \cdot \bar{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho)$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} (2\rho^2 Z^2) = \frac{1}{\cancel{\rho}} \cancel{\rho} \, 4Z^2$$

$$\int_V (\nabla \cdot D)\, dv$$

$$= \int_{z=-1}^{1} \int_{\phi=0}^{2\pi} \int_{\rho=0}^{5} 4z^2 \; \rho \, d\rho \, d\phi \, dz$$

$$= \frac{4z^3}{3} \Big|_{-1}^{1} * \frac{\rho^2}{2} \Big|_{0}^{5} * 2\pi$$

From (a) & (b)

$$= \frac{200\pi}{3}$$

Verification of Div. Theory #

$$\oint \bar{D} \cdot d\bar{s} = \int_V (\bar{\nabla} \cdot \bar{D})\, dV$$

surface integeral   Volume integeral

Electrical Power and Machines Depart.
Electromagnetic Fields
Sheet (3)
Coulombs Law and Electric Field Intensity

Tanta University — Faculty of Engineering

1. A 20 nC point charge is located at P (2, 4,-3) in free space. **Find**
   (a) **E** (r).
   (b) **E** at A (-3,2,0).

2. Point charges Q_1 = 5 μC and Q_2 = - 4 μC are placed at (3, 2, 1) and (-4, 0, 6), respectively.
Determine the force on Q_1.

3. Point charges Q_1 and Q_2 are, respectively, located at (4, 0, -3) and (2, 0, 1). If Q_2 = 4 nC, **Find** Q_1 such that
(a) The **E** at (5, 0, 6) has no z-component
(b) The force on a test charge at (5, 0, 6) has no x-component.

4. **Determine** the total charge
(a) On line $0 < x < 5$ m if $\rho_l = 12x^2$ mC/m
(b) On the cylinder $p = 3$, $0 < z < 4$ m if $\rho_s = \rho z^2$ nC/m^2
(c) Within the sphere r = 4 m if $\rho_v = \frac{10}{r \sin\theta}$ C/m^3

5. A line charge density ρ_l is uniformly distributed over a length of 2a with centre as origin along x axis. **Find E** at a point P which is on z axis at a distance d.

6. It is required to hold four equal point charges each in equilibrium at the corners of square. **Find** the point charge which will do this, if placed at the centroid of the square.

7. A ring placed along $y^2 + z^2 = 4$, $x = 0$ carries a uniform charge of 5 μC/m.
(a) **Find E** at P (3,0, 0).
(b) If two identical point charges Q are placed at (0, -3, 0) and (0, 3, 0) in addition to the ring,
   Find the value of Q such that **E** = 0 at P.

8. A sheet of charges ρ_s = 2 nC /m^2, is present at the plane $x = 3$ in free space, and a line charge ρ_l = 2 nC 1m 1 is located at $x = 1, z = 4$. Find
   (a) The magnitude of electric field intensity at the origin.
   (b) The direction of **E** at p (4, 5, 6).
   (c) What is the force per meter length on the line charge?

9. A point charge 100 pC is located at (4, 1,-3) while the x-axis carries charge 2 nC/m. If the plane z = 3 also carries charge 5 nC/m^2, **Find E** at (1, 1, 1).

Sheet ③

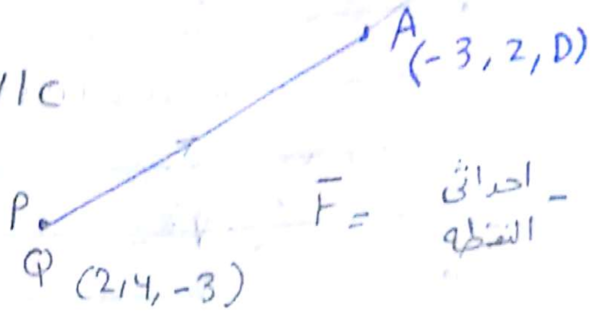
Ex ①: given: $Q = 20\,nC$ at $P(2,4,-3)$

Required a) $E(r)$ b) E at $A(-3,2,0)$

Solution

a) $\bar{E} = \dfrac{Q}{4\pi\varepsilon r^2}\,\bar{a}_r \quad N/C$

$\bar{E} = \dfrac{20\times10^{-9}}{4\pi\varepsilon_0 r^2}\,\bar{a}_r$



$\bar{F} =$ احداثي النقطة - احداثي مصدر المجال

b) at $A(-3,2,0)$

$\bar{r}_{PA} = A - P = -5\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z$

$\therefore |\bar{r}_{PA}| = 6.164$

$\therefore \bar{E} = -3.84\,\bar{a}_x - 1.53\,\bar{a}_y + 2.30\,\bar{a}_z$

Ex ③: given: Q_1 at $(4,0,-3)$ A

Q_2 at $B(2,0,1)$, $= 4\,nC$

Find Q_1 ? such that:

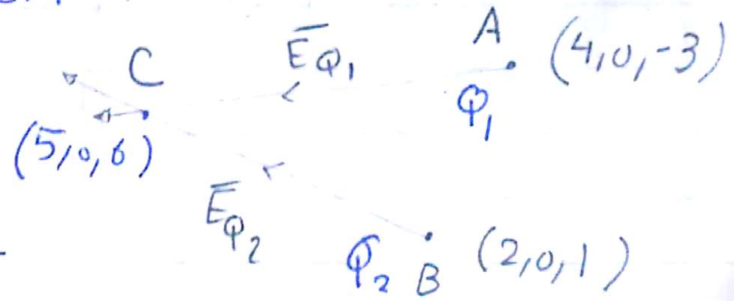
حل الطلاب a) $\bar{E}_z$ at $C(5,0,6) = \text{Zero}$

b) $\bar{F}_x$ on q_t at $(5,0,6) = \text{Zero}$

test charge

Solution

$\bar{E}_{tot} = \bar{E}_{Q_1} + \bar{E}_{Q_2}$



$= \dfrac{Q_1}{4\pi\varepsilon_0 R_{AC}^3}\,\bar{R}_{AC} + \dfrac{Q_2}{4\pi\varepsilon_0 R_{BC}^2}\,\bar{R}_{BC}$

$\bar{R}_{AC} = C - A = 11\,\bar{a}_x + 9\,\bar{a}_z \;,\; |\bar{R}_{AC}| = \sqrt{82}$

$\bar{R}_{BC} = C - B = 3\,\bar{a}_x + 5\,\bar{a}_z \;,\; |\bar{R}_{BC}| = \sqrt{34}$

$$\bar{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{Q_1}{82\sqrt{82}} + \frac{3 * 4 * 10^{-9}}{34\sqrt{34}}\right)\bar{a}_x + \left(\frac{9Q_1}{82\sqrt{82}} + \frac{5 * 4 * 10^{-9}}{34\sqrt{34}}\right)\right]\bar{a}_z \rightarrow (*)$$

for E_Z = Zero

E_Z

$$\therefore \frac{9Q_1}{82\sqrt{82}} = -\frac{5 * 4 * 10^{-9}}{34 * \sqrt{34}}$$

$$\therefore Q_1 = -8.32 \text{ nC}$$

for F_x = Zero on test charge q

$\because F_x$ = Zero $\Rightarrow \therefore E_x = \frac{F_x}{q}$ = Zero

from $(*)$

$$\therefore \frac{Q_1}{82\sqrt{82}} = -\frac{3 * 4 * 10^{-9}}{34 * \sqrt{34}}$$

$$\therefore Q_1 = -0.499 \text{ nC}$$

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$$Q = \int \rho \, d\varepsilon$$

a) $Q_l = \int \rho_L \, dl = \int_{x=0}^{5} 12 x^2 \, dx$

$$= \frac{12x^3}{3} \Big|_0^5 = 0.5 \, C$$

b) $Q_s = \int\int \rho_s \, ds = \int_{z=0}^{4} \int_{\phi=0}^{2\pi} \rho z^2 \Big|_{\rho=3} \rho \, d\phi \, dz$

$$= 9 \; \frac{z^3}{3} \Big|_0^4 * \phi \Big|_0^{2\pi}$$

$$= 1.206 \; \mu C$$

c) $Q_v = \int \int \int \rho_v \, dv$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{r=0}^{4} \frac{10}{\cancel{r \sin\theta}} r^{\cancel{2}} \cancel{\sin\theta} \, dr \, d\theta \, d\phi$$

$$10 * \frac{r^2}{2} \Big|_0^4 * \theta \Big|_0^{\pi} * \phi \Big|_0^{2\pi}$$

$$= 157.9 \; C$$

Ex ②: given: $Q_1 = 5\mu C$ at $(3,2,1)$
$Q_2 = -4\mu C$ at $(-4,0,6)$
Find F_{Q_1} ?

Solution

$$\bar{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\,\bar{a}_R = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^3}\,\bar{R}$$

$\bar{F}_{Q_1}$
Q_1 $(3,2,1)$

$$\bar{R}_{21} = r_1 - r_2$$

Q_2 $(-4,0,6)$

$$= 7\bar{a}_x + 2\bar{a}_y - 5\bar{a}_z$$

$\bar{R}$ = احداثي الشحنة المدروسة - احداثي مصدر القوه

$$|\bar{R}_{21}| = \sqrt{78}$$

$\bar{F}_{21}$ القوه المؤثره على Q_1 نتيجة Q_2

$$\therefore \bar{F}_{Q_1} = \frac{5\times10^{-6} \times -4\times10^{-6}}{4\pi \times 8.85\times10^{-12}\,(\sqrt{78})^3}\,(7\bar{a}_x - 2\bar{a}_y - 5\bar{a}_z)$$

$$= -1.827\bar{a}_x - 0.522\bar{a}_y + 1.31\bar{a}_z \quad mN$$

Ex ④: given

a) $\rho_L \quad 0 < x < 5 \quad = 12x^2 \; C/m$

b) $\rho_s = \rho Z^2 \; nC/m^2 \quad \rho = 3, \quad 0 \leq Z < 4$

c) $\rho_v = \dfrac{10}{r\sin\theta} \; C/m^2, \quad r = 4$

Solution

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$$\bar{E}_{Qtot} = \frac{Q}{4\pi\varepsilon_0 r_+^2}\bar{a}_{r+} + \frac{Q}{4\pi\varepsilon_0 r_-^2}\bar{a}_{r-}$$

$$\bar{r}_+ = \cancel{-3\bar{a}_y} + 3\bar{a}_z \qquad r = \sqrt{18}$$

$$\bar{r}_- = \cancel{3\bar{a}_y} + 3\bar{a}_z$$

$$\therefore \bar{E}_{Qtot} = \frac{Q}{4\pi\varepsilon_0 \sqrt{18}^3} 6\,\bar{a}_z$$

$$\therefore \frac{-0.32}{\cancel{\varepsilon_0}} = \frac{6Q}{4\pi\cancel{\varepsilon_0}\sqrt{18}^3}$$

$$\therefore Q = -51.1\ \mu C$$

Ex ⑦: given.

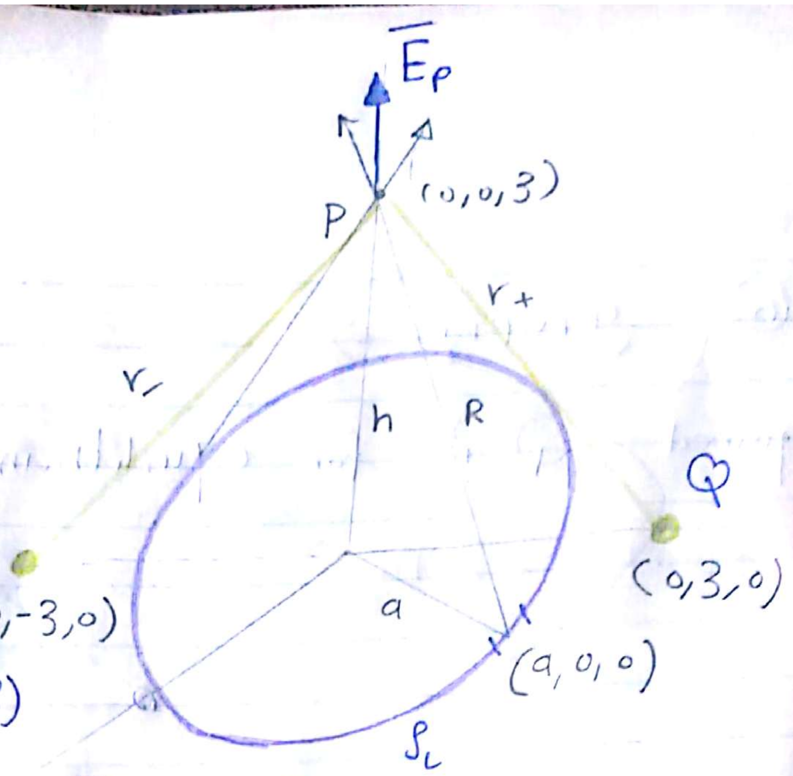
Ring $\cancel{Z} = 0$

$\cap\ y^2 + \cancel{X}^2 = 4$

$\rho_L = 5\ \mu C/m$

Find a) E_p $(0,0,3)$

b) Q ? $\Rightarrow$ $E_{tot_p} = \text{Zero}.$



Solution

a)

$$E = \int \frac{\rho_L\, dl}{4\pi\varepsilon_0 R^2}\ \bar{a}_R$$

For summerty

$E_\rho = \text{zero}$ $\therefore \bar{E} = \bar{E}_z$

$|\bar{R}| = \sqrt{13}$

$\bar{R} = -2\bar{a}_\rho + 3\bar{a}_z$

$dl = a\, d\phi$

$$\therefore \bar{E} = \frac{5\times10^{-6} \cancel{\times 2} \times 3}{2\,\cancel{4}\pi\varepsilon_0 (13)^{3/2}} \quad \cancel{\int_0^{2\pi} d\phi}^{\,2\pi}\ \bar{a}_z$$

$$\bar{E}_{Ring} = \frac{0.32}{\varepsilon_0}\ \mu N/C$$

b) $E_{tot} = \text{Zero}$

$= E_{Ring} + E_{Q_{tot}}$

$= E_{Ring} + \bar{E}_{+r} + \bar{E}_{-r}$

1. A point charge Q = 100 μC is located at the origin. **Determine** the total flux passing through the following surfaces:

(a) A hemisphere of radius 2m defined by $0 \le \theta \le 0.5\pi$, $0 \le \phi \le 2\pi$, $r = R_A$

(b) A spherical shell defined by $\theta_1 \le \theta \le \theta_2$

2. Cylindrical surfaces at r = 2, 4, and 6m carry uniform charge densities of 20, -4, and ρ_{so} nC/m^2 respectively. **Find**

(a) **D** at r = 1, 3, and 5 m

(b) **How much** electric flux passes through the closed surface $\rho = 8$ m, $0 < z < 1$?

(c) $\boldsymbol{\rho_s}$ such that **D** = 0 at r = 7 m.

3. Given that

$$\rho_v = \begin{cases} 12\rho \text{ nC/m}^3, & 1 < \rho < 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine **D** everywhere.

4. The Cylindrical surfaces at $\rho = 1, 2,$ and 3m carry uniform surface charge densities of 20, -8, and 5 nC/ m^2 respectively.

(a) **How much** electric flux passes through the closed surface $\rho = 5$, $0 < z < 1$?

(b) **Find D** at $\rho = 1, 3,$ and 5 m.

5. Let

$$\rho_v = \begin{cases} \dfrac{10}{r^2} \text{ mC/m}^3, & 1 < r < 4 \\ 0, & r > 0 \end{cases}$$

(a) Find the net flux crossing surface $r = 2$ m and $r = 6$ m.

(b) Determine **D** at $r = 1$ m and $r = 5$ m.

6. A uniform volume of 80 μC/m^3, is present throughout the region 8mm < r < 10mm. Let $\rho_v = 0$ for 0 < r < 8mm. **Find**

(a) The total charge inside the spherical surface r = 10mm.

(b) $\mathbf{D_r}$ at r = 10mm.

(c) If there is no charge for r >10mm, find $\mathbf{D_r}$ at r = 20mm.

7. In the region of free space that incudes the volume, $2 < x, y, z < 3$,

$$\mathbf{D} = \frac{2}{z^2}\left(yz\,\mathbf{a_x} + xz\,\mathbf{a_y} - 2xz\,\mathbf{a_z}\right) \text{ C/ m}^2.$$

(a) **Evaluate** the volume integral side of the divergence theorem for the volume defined by $2 < x, y, z < 3$.

(b) **Evaluate** the surface integral side for the corresponding closed surface.

8. If $\mathbf{D} = \left(15\,\rho^2 \sin 2\phi\ \mathbf{a}_\rho + 20\,\rho^2 \cos 2\phi\ \mathbf{a}_\phi\right)$ C/ m^2. **Evaluate** both sides of the divergence theorem for the region $1 < \rho < 2$m, $1 < \phi < 2$rad, $1 < z < 2$m.

9. If $\mathbf{D} = \frac{16}{r} \cos 2\theta\ \mathbf{a}_\theta$ C/ m^2. **Use** two different methods to **find** Q_{tot} within the region $1 < r < 2$m, $1 < \phi < 2$rad, $1 < \theta < 2$rad.

1. A point charge Q = 100 μC is located at the origin. **Determine** the total flux passing through the following surfaces:

(a) A hemisphere of radius 2m defined by $0 \le \theta \le 0.5\pi$, $0 \le \phi \le 2\pi$, $r = R_A$

(b) A spherical shell defined by $\theta_1 \le \theta \le \theta_2$

2. Cylindrical surfaces at r = 2, 4, and 6m carry uniform charge densities of 20, -4, and ρ_{so} nC/m^2 respectively. **Find**

(a) **D** at r = 1, 3, and 5 m

(b) **How much** electric flux passes through the closed surface $\rho = 8$ m, $0 < z < 1$?

(c) $\boldsymbol{\rho_s}$ such that **D** = 0 at r = 7 m.

3. Given that

$$\rho_v = \begin{cases} 12\rho \text{ nC/m}^3, & 1 < \rho < 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine **D** everywhere.

4. The Cylindrical surfaces at ρ = 1, 2, and 3m carry uniform surface charge densities of 20, -8, and 5 nC/ m^2 respectively.

(a) **How much** electric flux passes through the closed surface $\rho = 5$, $0 < z < 1$?

(b) **Find D** at ρ = 1, 3, and 5 m.

5. Let

$$\rho_v = \begin{cases} \dfrac{10}{r^2} \text{ mC/m}^3, & 1 < r < 4 \\ 0, & r > 0 \end{cases}$$

(a) Find the net flux crossing surface $r = 2$ m and $r = 6$ m.

(b) Determine **D** at $r = 1$ m and $r = 5$ m.

6. A uniform volume of 80 μC/m^3, is present throughout the region 8mm < r < 10mm. Let $\rho_v = 0$ for 0 < r < 8mm. **Find**

(a) The total charge inside the spherical surface r = 10mm.

(b) $\mathbf{D_r}$ at r = 10mm.

(c) If there is no charge for r >10mm, find $\mathbf{D_r}$ at r = 20mm.

7. In the region of free space that incudes the volume, $2 < x, y, z < 3$,

$$\mathbf{D} = \frac{2}{z^2}\left(yz\,\mathbf{a_x} + xz\,\mathbf{a_y} - 2xz\,\mathbf{a_z}\right) \text{ C/ m}^2.$$

(a) **Evaluate** the volume integral side of the divergence theorem for the volume defined by $2 < x, y, z < 3$.

(b) **Evaluate** the surface integral side for the corresponding closed surface.

8. If $\mathbf{D} = \left(15\,\rho^2 \sin 2\phi\ \mathbf{a_\rho} + 20\,\rho^2 \cos 2\phi\ \mathbf{a_\phi}\right)$ C/ m². **Evaluate** both sides of the divergence theorem for the region $1 < \rho < 2$m, $1 < \phi < 2$rad, $1 < z < 2$m.

9. If $\mathbf{D} = \frac{16}{r} \cos 2\theta\ \mathbf{a_\theta}$ C/ m². **Use** two different methods to **find** Q_{tot} within the region $1 < r < 2$m, $1 < \phi < 2$rad, $1 < \theta < 2$rad.

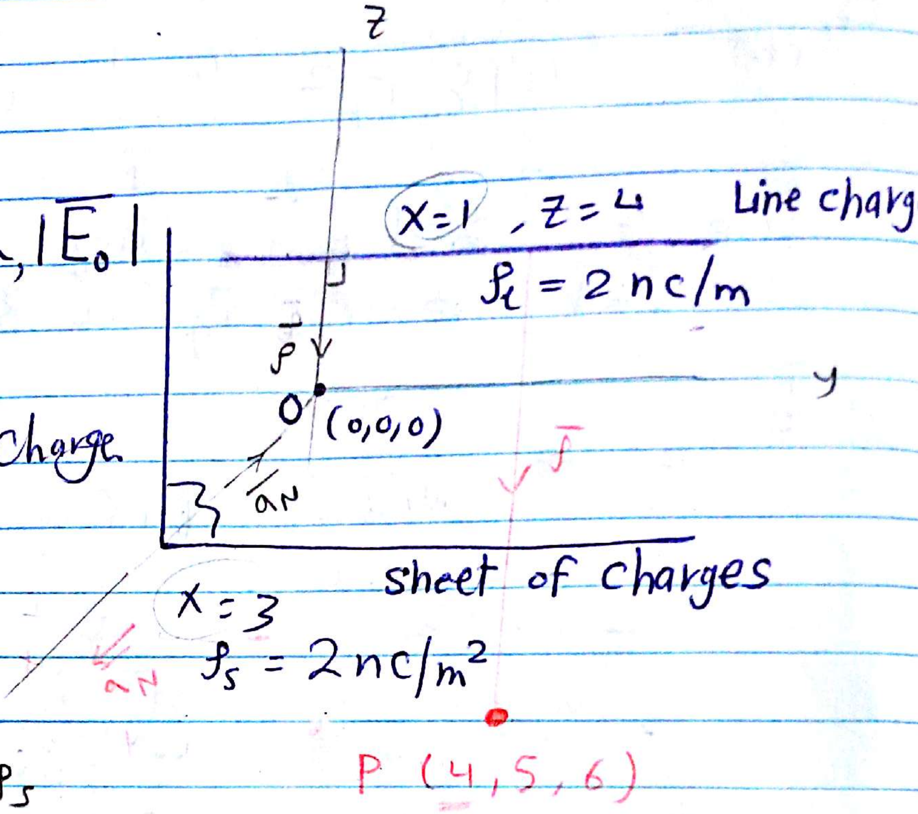
Ex [8]: given:

Find:

a) $|\bar{E}|$ at origin, $|E_o|$

b) $\bar{E}_p$, $P(4,5,6)$

c) F/l on line charge



Solution

a) $\bar{E}_{tot} = \bar{E}_{\rho_l} + \bar{E}_{\rho_s}$

$$= \frac{\rho_l}{2\pi\varepsilon_o \rho^2}\,\bar{\rho} + \frac{\rho_s}{2\varepsilon_o}\,\bar{a}_N$$

$$\bar{\rho} = (0,0,0) - (1, y, 4) = (-1, 0, -4)$$

$$\rho = \sqrt{17}$$

$$\bar{a}_N = -\bar{a}_x$$

$$\therefore \bar{E}_{tot} = \frac{2\times 10^{-9}}{2\pi\varepsilon_o (17)}(-1,0,4) + \frac{2\times 10^{-9}}{2\varepsilon_o}(-1,0,0)$$

$$= -115\,\bar{a}_x + 8.45\,\bar{a}_z$$

$$|\bar{E}_o| = \sqrt{} = 115.31\ N/C$$

$$\bar{E}_{Q_{tot}} = \frac{Q}{4\pi\varepsilon_0 r_+^2}\,\bar{a}_{r_+} + \frac{Q}{4\pi\varepsilon_0 r_-^2}\,\bar{a}_{r_-}$$

$$\bar{r}_+ = \cancel{-3\bar{a}_y} + 3\bar{a}_z \qquad r = \sqrt{18}$$

$$\bar{r}_- = \cancel{3\bar{a}_y} + 3\bar{a}_z$$

$$\therefore \bar{E}_{Q_{tot}} = \frac{Q}{4\pi\varepsilon_0 \,{}^{3}\!\sqrt{18}}\; 6\,\bar{a}_z$$

$$\therefore \frac{-0.32}{\cancel{\varepsilon_0}} = \frac{6Q}{4\pi\cancel{\varepsilon_0}\,{}^{3}\!\sqrt{18}}$$

$$\therefore Q = -51.1\ \mu C$$

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b) $\bar{\rho} = (4,5,6) - (1,y,4) = (3,0,2)$

$|\bar{\rho}| = \sqrt{15}$

$\bar{a}_N = + \bar{a}_x$

$\therefore \bar{E}_p = 120.13\, \bar{a}_x + 4.79\, \bar{a}_z$

c)

القوه المؤثره على شحنة $\rightarrow \bar{F} = q\, \bar{E} \leftarrow$ المجال المتواجد فيه الشحنه

الشحنة المدروسة

الشحنه المدروسة ← خط الشحنات

$q = \rho_L\, l$

مصدر المجال ≡ مصدر القوه ← سطح الشحنات

$\bar{E}_{\rho_s} = \frac{\rho_s}{2\varepsilon_0}\, \bar{a}_N$

$\bar{a}_N = -\bar{a}_x$

$\frac{F}{l} = \rho_L \frac{\rho_s}{2\varepsilon_0} - \bar{a}_x$

$= 2.27 \times 10^{-7}$ N/m

Ex $\boxed{9}$: given:

$Q = 100 \, PC$ at $(4,1,-3)$

$\rho_L = 2 nc/m$ on x - axis $(y=0, Z=0)$

$\rho_s = 5 nc/m^2$ on Z - plane at $Z = 3$

Find:

$\bar{E}_A \rightarrow (1,1,1)$

solution

$$\bar{E}_{tot} = \frac{Q}{4\pi\varepsilon_0 R^3} \bar{R} + \frac{\rho_L}{2\pi\varepsilon_0 \rho^2} \bar{\rho} + \frac{\rho_s}{2\varepsilon_0} \bar{a}_N$$

$\bar{R} = (1,1,1) - (4,1,-3) = (-3,0,4)$

$\bar{\rho} = (1,1,1) - (1,0,0) = (0,1,1)$

$\bar{a}_N = -\bar{a}_Z \qquad Z_{\rho_s} > Z_{point}$

$$= \frac{100 * 10^{-12}}{4\pi\varepsilon_0 (25)^{3/2}} (-3,0,4) + \frac{2 * 10^{-9}}{2\pi\varepsilon_0 (2)} (0,1,1)$$

$$+ \frac{5 * 10^{-9}}{2\varepsilon_0} (0,0,-1)$$

Ex ①: given Q at $(0,0,0)$

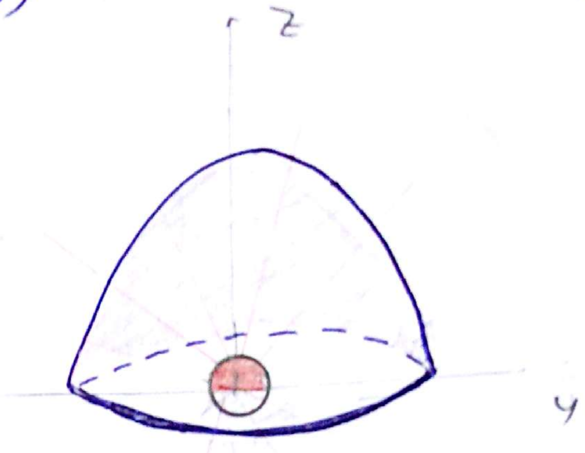
Find Ψ_{tot}

a) hemisphere, $r=2$

$0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq 2\pi$

b) spherical shell

$\theta_1 \leq \theta \leq \theta_2$, $r = R$



$Q = 100 \mu C$

Solution

a) $\Psi = \oint_s \bar{D}_r \cdot \bar{ds}_r = Q_{enc} = Q/2$ (From Gauss)

$\bar{D}_r = \dfrac{Q}{4\pi\varepsilon_0 r^2} \bar{a}_r$, $\bar{ds}_r = r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$

$\therefore \quad = \dfrac{Q \cancel{r^2}}{2\,\cancel{4\pi\varepsilon_0 r^2}} \displaystyle\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \sin\theta \, d\theta \, d\phi$ $\quad \cancel{2\pi}$

$\Psi = \dfrac{Q}{2} = 50 \, \mu wb$

b) $\Psi = \dfrac{Q}{2\,\cancel{4\pi\varepsilon_0 r^2}} \cancel{r^2} \displaystyle\int_{\phi=0}^{2\pi}\int_{\theta=\theta_1}^{\theta_2} \sin\theta \, d\theta \, d\phi$

$(\cos\theta_1 - \cos\theta_2) \cancel{2\pi}$

$= \dfrac{Q}{2} (\cos\theta_1 - \cos\theta_2)$

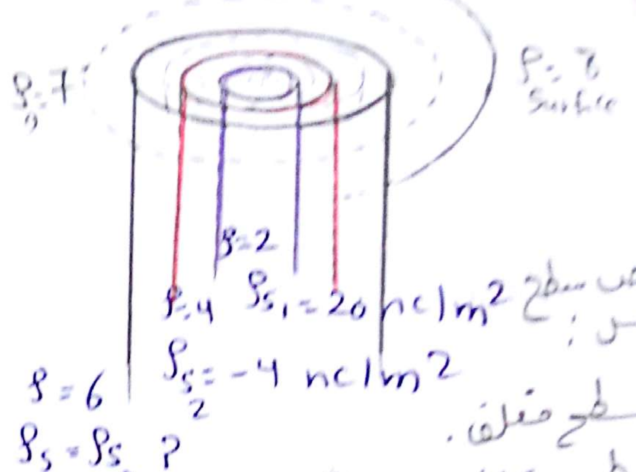
$\Psi = 50 (\cos\theta_1 - \cos\theta_2) \, \mu wb$

EX(2): given: cylindrical surfaces

Req: a) $\bar{D}$ at $\rho = 1, 3, 5$ m

b) Ψ pass $\rho_{s_3} = 9\,nC/m^2$, $\rho = 8$ m, $0 < Z < 1$

c) $\rho_{s_0} \rightarrow D = 0$) $\rho = 7$m



خواص سطح جاوس :

1 - سطح مغلق .

2 - سطح متماثل حول الشحنة

3 - يمر بالنقطة المدروسة .

solution

Gauss law

$$\oint D \cdot ds = Q_{enc}$$

الشحنة الكلية الموجودة داخل سطح جاوس

كثافة المجال الكهربي

سطح جاوس

a) at $\rho_g = 1$m $\rightarrow$ $Q_{enc} = $ zero $\therefore D = 0$

at $\rho_g = 3$ m $\rightarrow$ $Q_{enc} = Q_1 = \rho_{s_1} A_{1\,side} = 2\pi\rho_1 L \rho_{s_1}$

$$\oint \bar{D}_\rho \cdot d\bar{s}_\rho = D_\rho \int_{\phi=0}^{2\pi}\int_{Z=0}^{\rho} r\, d\phi\, dZ = D(2\pi\rho_g L)$$

$$\therefore D(\cancel{2\pi}\, \overset{3}{\rho_g}\, \cancel{L}) = \overset{20nC/m^2}{\rho_{s_1}}\, \cancel{2\pi}\, \overset{2}{\rho_1}\, \cancel{L}$$

$$\therefore \bar{D}\Big|_{\rho=3} = \frac{2 * 20 * 10^{-9}}{3} = 13.3 * 10^{-9}\ \bar{a}_r\ wb/m^2$$

at $\rho_g = 5$ m $\rightarrow$ $Q_{enc} = Q_1 + Q_2 = \rho_{s_1} A_1 + \rho_{s_2} A_2$

$$= \cancel{2\pi}L(\overset{2}{\rho_1}\rho_{s_1} + \rho_2 \overset{-4}{\rho_{s_2}})$$

$$\oint \bar{D}_\rho \cdot d\bar{s}_\rho = D(\cancel{2\pi}\, \overset{5}{\rho_g}\, L)$$

$$\therefore D = \frac{2 * 20 * 10^{-9} - 4 * 4 * 10^{-9}}{5} = 4.8 * 10^{-9}\ \bar{a}_r\ wb/m^2$$

b) $\because \Psi = Q_{enc} = {}_s\oint D_\rho \cdot dS_\rho$?

$= Q_1 + Q_2 + Q_3$

$= 2\pi l \, (\rho_1 \rho_{s_1} + \rho_2 \rho_{s_2} + \rho_3 \rho_{s_3})$

$= 2\pi * 1 * 10^{-9} \, (2 * 20 - 4 * 4 + 6 * 4)$

$= 3.76 * 10^{-7}$ wb

c) ρ_{s_0} ? From Gauss

$\oint \bar{D} \cdot d\bar{S} = Q_{enc} = \cancel{2\pi} \cancel{l} (\rho_1 \rho_{s_1} + \rho_2 \rho_{s_2} + \rho_{s_3} \rho_{s_0})$

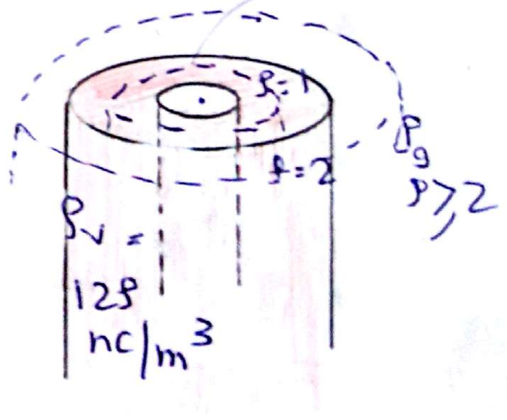
$\cancel{D} \cancel{(2\pi \rho_g l)} = (2 * 20 * 10^{-9} - 4 * 4 * 10^{-9} + 6 \rho_{s_0})$

$\therefore \rho_{s_0} = -4 \, nc/m^2$

Ex ③: given: $\rho_v = \begin{cases} 12\rho \; nc/m^3 & 1 < \rho < 2 \\ 0 & \text{otherwise} \end{cases}$

Determine $\bar{D}$ everywhere.

Solution.



everywhere

$\rho < 1$

$Q_{enc} = $ Zero

$\therefore D = $ Zero $\quad \rho < 1$

$1 < \rho < 2$

$Q_{enc} = Q_{volume}$

$\bar{D}$?

$\rho > 1$

$Q_{enc} = Q_{tot}$

$\bar{D}$?

Ex ③ $1 \le \rho < 2$

$$Q_{enc} = \int_v \rho_v \, dv = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \int_{\rho=1}^{\rho_g} 12 \rho^2 \, d\rho \, d\phi \, dz$$

$$= \cancel{12}^{\,4} \frac{\rho^3}{\cancel{3}} \Big|_{\rho=1}^{\rho_g} * 2\pi * L = \underset{4}{\cancel{8}} \pi \cancel{L} (\rho_g^3 - 1) \rightarrow ①$$

$$\oint_s D_\rho \cdot ds_\rho = D_\rho \int_0^L \int_{\phi=0}^{2\pi} \rho_g \, d\phi \, dz$$

$$= D (\cancel{2\pi} \rho_g \cancel{L}) \rightarrow ②$$

$$\therefore \underset{0 < \rho_g < \rho}{\bar{D}} = \frac{4(\rho^3 - 1)}{\rho_g} \bar{a}_\rho$$

$\rho \geq 2$

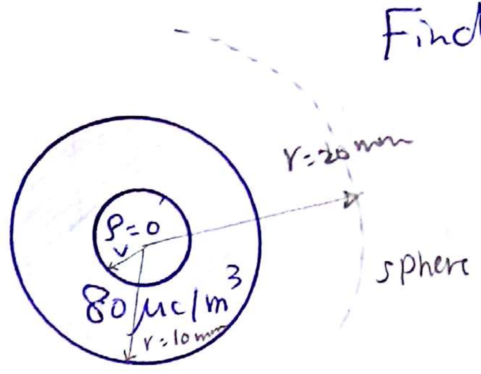
$$Q_{enc} = Q_{tot} = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \int_{\rho=1}^{2} 12 \rho^2 \, d\rho \, d\phi \, dz$$

$$= 8\pi L (2^3 - 1^3) = \underset{28}{\cancel{56}} \pi \cancel{L}$$

$$\oint D_\rho \cdot ds_\rho = D (\cancel{2\pi} \rho_g L)$$

$$\underset{\rho \geq 2}{\bar{D}} = \frac{28}{\rho_g} \bar{a}_\rho$$

Ex 6: given.



$8 < r < 10$ mm

Find: a) Q_{tot} inside sphere $r = 10$ mm

b) $\bar{D}_r$ at $r = 10$ mm

c) D_r at $r = 20$ mm

if $Q = 0$ for $r > 10$ mm

Solution

a) $$Q_{tot} = \cancel{Q_{0\to 8}} + Q_{8\to 10} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=8}^{r=10} \rho_v\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \rho_v \left.\frac{r^3}{3}\right|_8^{10} (2)(2\pi) = 5.45\ \text{PC}$$

b) From Gauss at $r = 10$ mm

$$\oint D_r \cdot ds_r = Q_{enc}$$

$$D_r \underbrace{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2 \sin\theta\, d\theta\, d\phi}_{(4\pi r^2)}$$

$$\therefore \bar{D}_r = \frac{Q_{enc}}{4\pi r^2} = \frac{5.45 \times 10^{-12}}{4\pi (10\times 10^{-3})^2}\, \bar{a}_r$$

$$= 4.4 \times 10^{-9}\, \bar{a}_r\ \text{wb/m}^2$$

c) at $r = 20$ mm

$$\bar{D}_r = \frac{5.45 \times 10^{-12}}{4\pi (20\times 10^{-3})^2}\, \bar{a}_r$$

$$= 1.102 \times 10^{-9}\, \bar{a}_r\ \text{wb/m}^2$$

1. **Find** the work done in carrying a 5 C charge from P (1, 2, -4) to R (3, -5, 6) in electric field

$$\mathbf{E} = \mathbf{a_x} + z^2\,\mathbf{a_y} + 2yz\,\mathbf{a_z}\ \text{V/m}$$

2. In free space, $V = x^2\, y\,(z+3)$ V. **Find**
   (a) **E** at (3, 4, -6)
   (b) **The charge** within the cube $0 < x, y, z < 1$

3. Using cylindrical coordinates, **Find** the work done in carrying a unit positive charge from $P_1\,(a, 0, 0)$ to $P_2\,(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}, 0)$ in electric field

$$\mathbf{E} = (r\sin\phi\ \cos\phi + z\cos\phi)\ \mathbf{a_\phi}\ \ \text{V/m}$$

4. Let $V = 2xy^2z^2$ V in free space. Evaluate each of the following quantities at P (3, 2, -1):
   (a) V; (b) **E**; (c) |**E**|; (b) $\mathbf{a_N}$; (c) **D**.

5. Given the potential field $V = 80\,r^2\text{Cos}\,\theta$ V and a point P (2.5, 30, 60) in free space, **Find** at P:
   (a) V; (b) **E**; (c) **D**; (d) ρ_v; (f) $\mathbf{a_N}$.

6. Given a surface charge density of 8 nC/m^2 on the plane x = 2, a line charge density of 30 nC/m on the line x = 1, y = 2 and 1 μC point charge at P (-1, -1, 2). **Find** V_{AB} for points A (3, 4, 0) and B (4, 0, 1).

7. A uniform volume charge density ρ_v C/m³ is present throughout a sphere with radius R. **Find** both **E** and V <u>inside</u> and <u>outside</u> the sphere. Note that there is no charge for $r > R$.

8. A finite line with uniformly distributed line charge density C/m. The line is located with centre as origin along z-axis. **Find** V and **E** at a point P with a distance r from the center of the line.

Sheet 4

$$= \frac{2y}{z}\bar{a}_x + \frac{2x}{z}\bar{a}_y + \frac{-4x}{z}\bar{a}_z$$

(with $D_x = \frac{2y}{z}$, $D_y = \frac{2x}{z}$, $D_z = \frac{-4x}{z}$)

$\boxed{7}$ $\bar{D} = \frac{2}{z^2}\left(yz\,\bar{a}_x + xz\,\bar{a}_y - 2xz\,\bar{a}_z\right)$

Req. → Vol. integral $\int \rho_v \, dv$ $\quad 2 < x, y, z < 3$

→ Surface integral $\oint D \cdot ds$

Solution.

Volume

$$\rho_v = \nabla \cdot D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \text{zero} + \text{zero} + \frac{4x}{z^2} = \frac{4x}{z^2}$$

$$\int \rho_v \, dv = \int_{z=2}^{3}\int_{y=2}^{3}\int_{x=2}^{3} \frac{4x}{z^2}\, dx\, dy\, dz$$

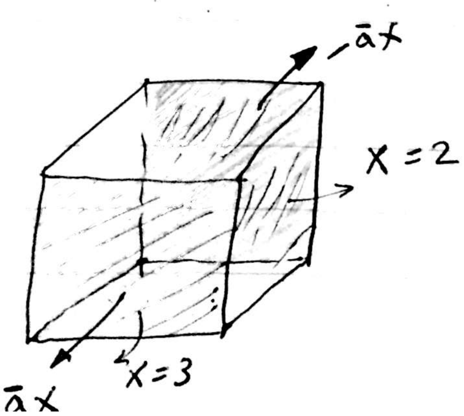
$$= \int_2^3 dy * \int_2^3 4x\, dx * \int_2^3 z^{-2} dz$$

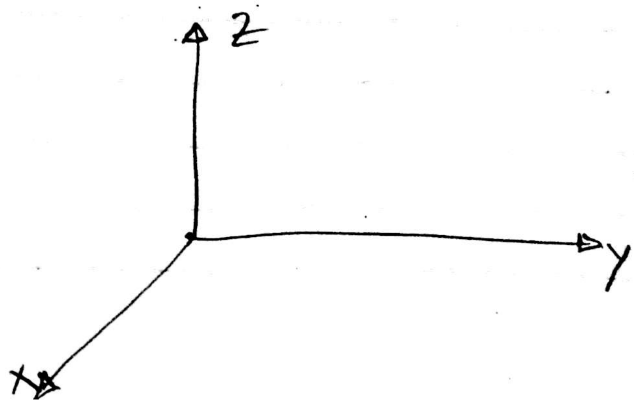
$$= (3-2) * 2x^2 \Big|_2^3 * \frac{z^{-1}}{-1}\Big|_2^3$$

$$= 1 * 2(9-4) * -1\left(\frac{1}{3} - \frac{1}{2}\right) \quad C$$

$$= 0.833 * 2 = 1.667 \quad \#$$

Surface integral $\oint D \cdot ds$ $\quad ds =$





$$ds \begin{cases} \to dx\,dy\,(\pm \bar{a}_z) \\ \to dx\,dz\,(\pm \bar{a}_y) \\ \to dy\,dz\,(\pm \bar{a}_x) \end{cases}$$

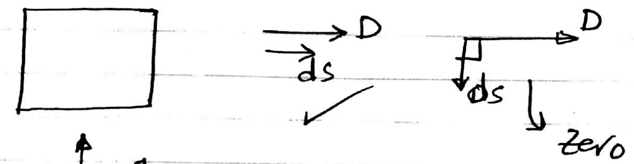
$+ \to x=3$

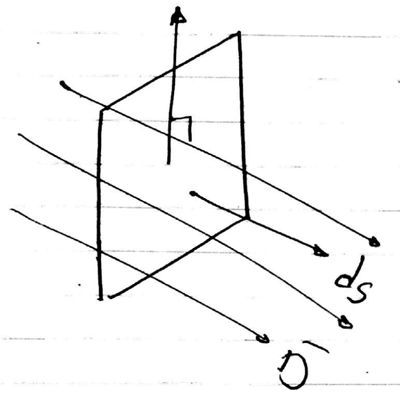
$- \to x=2$

المساحة ⊕ ← البعد الكبير

⊖ ← البعد الصغير

ملاحظة كل مساحة سيعبر منها مركبة D التي لها نفس اتجاه متجه المساحة





$\oint D \cdot ds$

سطح مغلق المكعب

① $\int_{x=2} -\bar{D} \cdot \bar{ds}_x$

$$= \int \frac{2}{z^2}\left(\underbrace{yz\,\bar{a}_x}_{} + \cancel{xz\,\bar{a}_y} - \cancel{2xz\,\bar{a}_z}\right) \cdot dy\,dz\,(-\bar{a}_x)$$

$$= \int_{x=2} -\bar{D}_x \; \bar{dS}_x$$

نكمل مع باقي الاسطح.

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على الأوجه الستة للمكعب

$$\oint D \cdot ds = \int_{①} + \int_{②} + \int_{③} + \int_{④} + \int_{⑤} + \int_{⑥}$$

① $\int_{x=2} -D_x \, dy\, dz = \int_{z=2}^{3}\int_{y=2}^{3} -\frac{2y}{z} \, dy\, dz$

$= -y^2 \Big|_2^3 * \ln z \Big|_2^3$

② $\int_{x=3} D_x \, dy\, dz = y^2 \Big|_2^3 * \ln z \Big|_2^3$

يلاشوا بعض

③ $\int_{y=2} -D_y \, dx\, dz = -\int_{z=2}^{3}\int_{x=2}^{3} \frac{2x}{z} \, dx\, dz$

④ $\int_{y=3} D_y \, dx\, dz = \int\int \frac{2x}{z} \, dx\, dz$

يلاشوا بعض

⑤ $\int_{z=2} -D_z \, dx\, dy = -\int_2^3\int_2^3 \frac{-4x}{2} \, dx\, dy$

$= \int\int 2x \, dx\, dy = x^2 \Big|_2^3 \; y \Big|_2^3$

$= (9-4)(3-2) = 5$

⑥ $\int_{z=3} D_z \, dx\, dy = \int_2^3\int_2^3 \frac{-4x}{3} \, dx\, dy = \frac{-2x^2}{3}\Big|_2^3 \; y\Big|_2^3$

$= \frac{-2(9-4)}{3} * 1 = \frac{-10}{3}$

$\oint D \cdot ds = 5 + \left(\frac{-10}{3}\right) = 1.667 \; C$ #

[9] $\bar{D} = \frac{16}{r}\cos 2\theta \; a_\theta \quad C/m^2$

Sol: $1<r<2$, $1<\phi<2$ rad, $1<\theta<2$

$Q_{tot} \rightarrow$ two methods

solution

method ①: $Q_t = \int_{vol.} (\nabla \cdot D)\, dv$

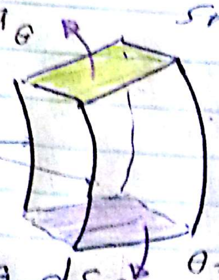
$$\nabla \cdot D = \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r)}_{0} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, D_\theta) + \underbrace{\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}}_{0}$$

$$= \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{16}{r}\cos 2\theta\right)$$

$$= \frac{16}{r^2\sin\theta}\left\{\sin\theta * (-2\sin 2\theta) + \cos 2\theta \cos\theta\right.$$

$$= \frac{16}{r^2}\left\{-2\sin 2\theta + \cos 2\theta \frac{\cos\theta}{\sin\theta}\right\}$$

method ②



$Q_{tot} = \oint \bar{D}\cdot d\bar{S}$

$$= \int_{\phi=1}^{2}\int_{r=1}^{2} -D_\theta \, dS_\theta \;(\theta=1) + \int_{\phi=1}^{2}\int_{r=2}^{2} D_\theta \, dS_\theta \;(\theta=2)$$

$dS_\theta = r\sin\theta\, dr\, d\phi$

Ex $\boxed{9}$ $\bar{D} = \frac{16}{r} \cos 2\theta \; a_\theta \quad C/m^2$

find: $1 < r < 2$, $1 < \phi < 2$ rad , $1 < \theta < 2$ rad

$Q_{tot} \rightarrow$ two methods

solution

method ①: $Q_t = \int_{vol.} (\nabla . D) \, dv$

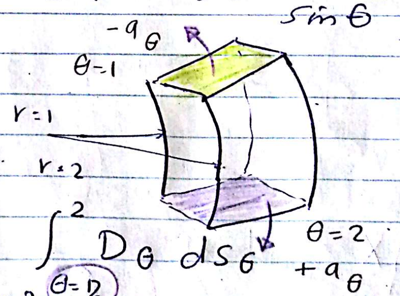
$$\nabla . D = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta \, D_\theta) + \frac{1}{r \sin\theta} \frac{\partial D_\phi}{\partial \phi}$$

$$= \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \, \frac{16}{r} \cos 2\theta \right)$$

$$= \frac{16}{r^2 \sin\theta} \left\{ \sin\theta * (-2 \sin 2\theta) + \cos 2\theta \cos\theta \right\}$$

$$= \frac{16}{r^2} \left\{ -2 \sin 2\theta + \cos 2\theta \frac{\cos\theta}{\sin\theta} \right\}$$

method ②



$$Q_{tot} = \oint \bar{D} . \bar{ds}$$

$$= \int_{1=\phi}^{2} \int_{r=1}^{2} -D_\theta \, ds_\theta \Big|_{\theta=1} + \int_{1=\phi}^{2} \int_{r=2}^{2} D_\theta \, ds_\theta \Big|_{\theta=2}$$

$$ds_\theta = r \sin\theta \, dr \, d\phi$$

#

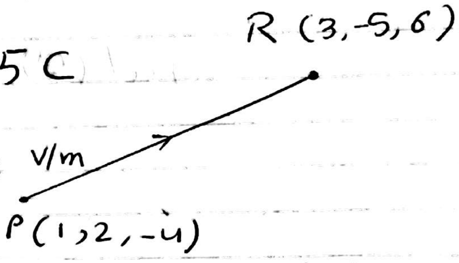
Sheet (5)

Sheet 5

Ex 1: given: $Q = 5C$

$\bar{E} = \bar{a}_x + z^2 \bar{a}_y + 2yz\, \bar{a}_z$ v/m

Req: $W_{P \to R}$



Solution

$$W_{P \to R} = -Q \int_{\text{initial}}^{\text{final}} \bar{E} \cdot d\bar{L}$$

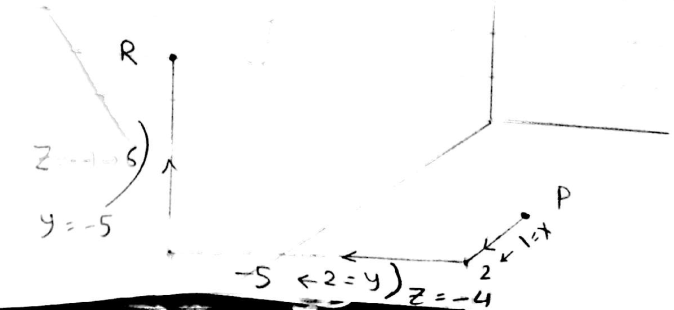
$$= 5 \int (\bar{a}_x + z^2 \bar{a}_y + 2yz\, \bar{a}_z) \cdot (dx\, \bar{a}_x + dy\, \bar{a}_y + dz\, \bar{a}_z)$$

$$= 5 \left(\int_1^3 dx + \int_2^{-5} z^2 \Big|_{z=-4} dy + \int_{-4}^{6} 2yz \Big|_{y=-5} dz \right)$$

$$= 5 \left(x \Big|_1^3 + z^2\, y \Big|_2^{-5} + 2y \frac{z^2}{2} \Big|_{-4}^{6} \right)$$

$$= 5 \left(2 + 16(-7) + (-5)(36 - 16) \right)$$

$$W = 1050\ J$$



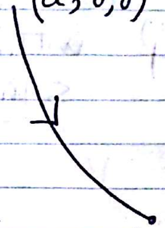
العكرمحفظ

Ex ③: given $\bar{E} = (\rho \sin\phi \cos\phi - Z\cos\phi)\,\bar{a}_\phi$

$Q = 1\,C$

Find $W_{P_1 \to P_2}$

using Cylindrical Coordinates.



Solution

$$W = -Q \int_{int}^{fin} \bar{E} \cdot d\bar{L}$$

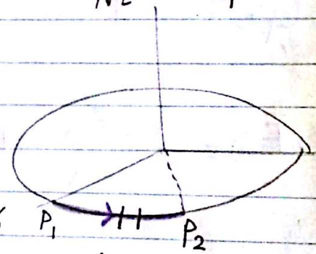
P_1 $(a,0,0) \rightarrow (\rho, \phi, Z) \rightarrow (a,0,0)$

$\rho = \sqrt{a^2 + 0} = a$, $\phi = \tan^{-1} \frac{0}{a} = 0$

P_2 $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}, 0\right) \rightarrow (\rho, \phi, Z) \rightarrow \left(a, \frac{\pi}{4}, 0\right)$

$\rho = \sqrt{\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{\sqrt{2}}\right)^2} = a$, $\phi = \tan^{-1} \frac{a/\sqrt{2}}{a/\sqrt{2}} = \frac{\pi}{4}$

$$\therefore W = -\int \bar{E}_\phi \cdot d\bar{L}_\phi$$



$$= -\int_0^{\pi/4} \left(r\Big|_{r=a} \sin\phi \cos\phi - Z\Big|_{Z=0} \cos\phi\right) a\,d\phi$$

$$= -a^2 \int^{\pi/4} \sin\phi \cos\phi \, d\phi$$

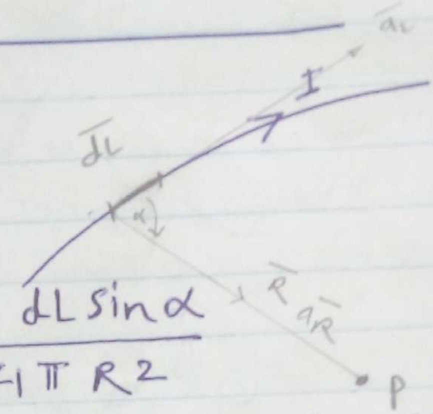
$$= -a^2 \int \frac{1}{2} \sin 2\phi \, d\phi = \frac{-a^2}{2}\left(-\frac{\cos 2\phi}{2}\Big|_0^{45}\right)$$

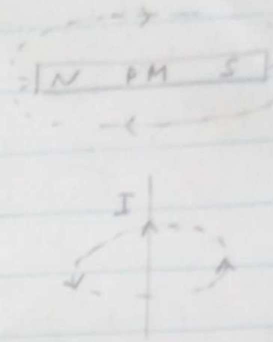
$$W = -\frac{a^2}{4} \;\; J$$

$\sin 2\theta = \frac{1}{2} \sin\theta \cos\theta$

CH 6 Steady magnetic field

Biot-Savart's law





$$\bar{dH}_p = \frac{I\,\bar{dL} \times \bar{R}}{4\pi R^3} = \frac{I\,dL \sin\alpha}{4\pi R^2}$$

$\bar{R}$ = احداثيات النقطة - احداثيات عنصر الطول

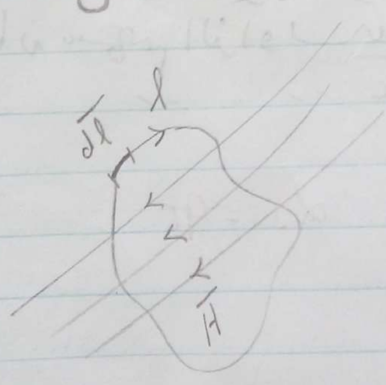
$$\boxed{\bar{H} = \frac{I}{4\pi} \int \frac{\bar{dL} \times \bar{R}}{R^3}}$$

, $\bar{B} = \mu \bar{H}$

Ampere's circuital law

$$\oint H \cdot dL = \Sigma I$$

حساب المجال المغناطيسي الناشئ من مرور تيار في موصل .



(2)

مجال كهربى $\bar{E}$

Electrostatic field الكتروستاتيكية

تختص بدراسة التأثير الكهربى للشحنات الساكنة

$\bar{H}$ $\bar{E}$

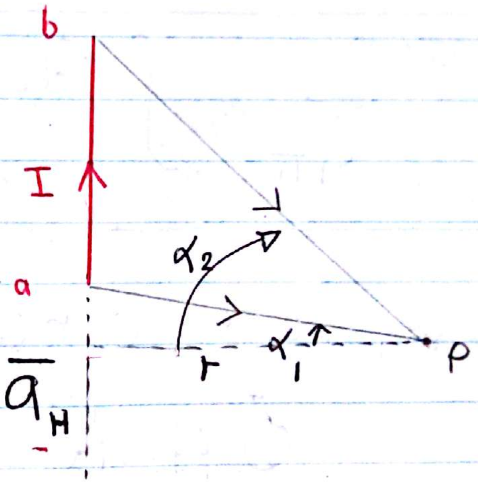
اما عندما نواجد كرمن التأثيرات الكهربية, والمغناطيسية فانه التفاعل بين الشحنات يوصف بدراسة → كهرومغناطيسية

Electromagnetic ما بين تواجد شحنات متحركة

تأثير مغناطيسي $(\bar{H})$

تأثير كهربى $(\bar{E})$

For Finit line.



$$\bar{a}_H = \bar{a}_I \times \bar{a}_r$$

$$\therefore \bar{H}_p = \frac{I}{4\pi r}(\sin\alpha_2 - \sin\alpha_1)\,\bar{a}_H$$

النهاية

البداية

البعد العمودى من النقطة للموصل

* لتحديد اشارة الزوايا:

- اذا كان سهم الزاوية مع سهم التيار ← الزاوية $\oplus$

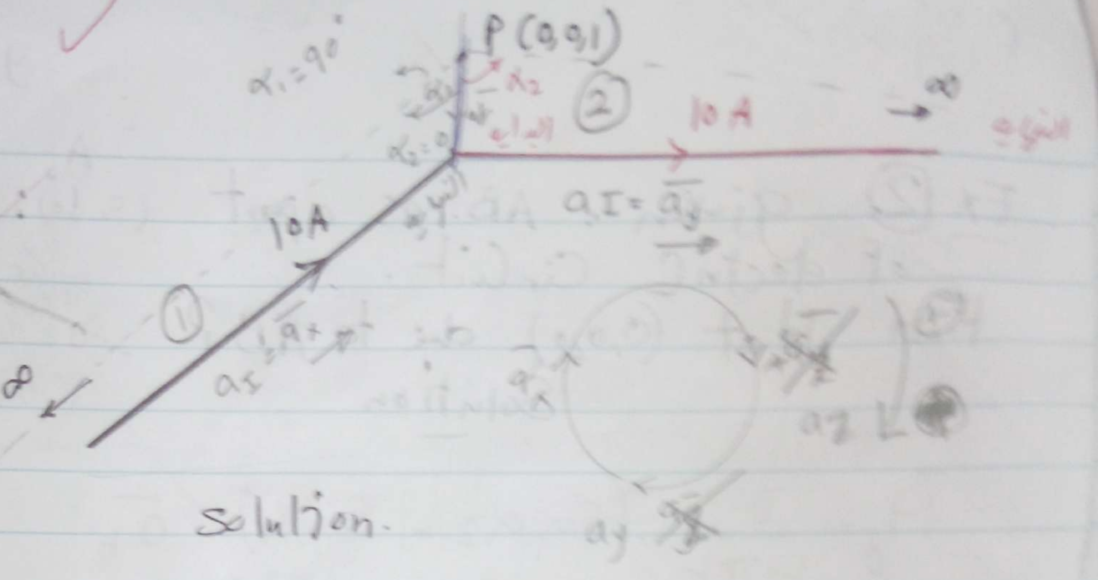
$\ominus$ ،، ،، عكس ،، ،، ← ،،

For infinit line $\alpha_1 = -90$, $\alpha_2 = 90$

$$\therefore \bar{H}_p = \frac{I}{2\pi r}\,\bar{a}_r$$

4   3

Ex 1 given:



Find H_p ?

solution.

$$\bar{H} = \frac{I}{4\pi r}(\sin\alpha_2 - \sin\alpha_1)\,\bar{a}_H$$

For Part 1:

$\alpha_1 = -90$

$\alpha_2 = 0$

$\bar{a}_H = \bar{a}_I \times \bar{a}_r$

$= -\bar{a}_x \times -\bar{a}_z$

$= +\bar{a}_y$

For Part 2:

$\alpha_1 = 0$

$\alpha_2 = 90°$

$\bar{a}_H = \bar{a}_I \times \bar{a}_r$

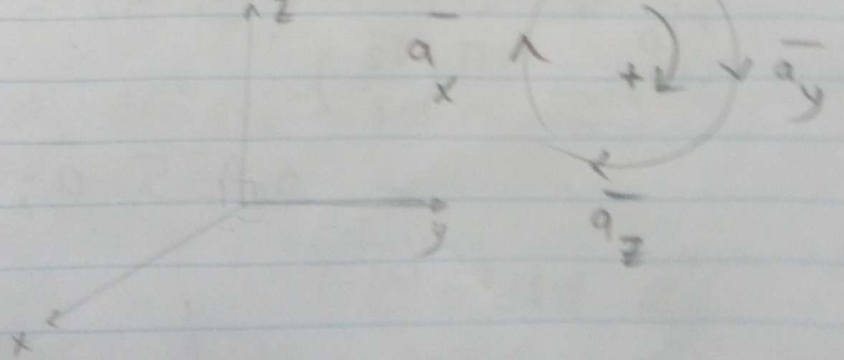
$= \bar{a}_y \times -\bar{a}_z$

$= -(-\bar{a}_x) = +\bar{a}_x$

$|r| = 1$, $I = 10A$

$$\therefore H_{p_1} = \frac{10}{4\pi(1)}\,\bar{a}_y \quad , \quad H_{p_2} = \frac{I}{4\pi(1)}\,\bar{a}_x$$

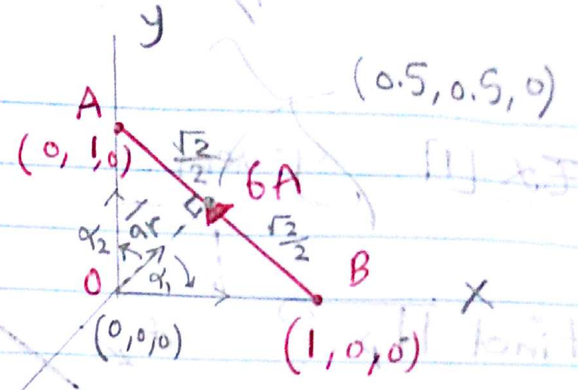
$$\therefore H_p = \frac{10}{4\pi}(\bar{a}_y + \bar{a}_x) \quad A/m$$



P 7.3 (pg. 303, S. pg 171)

4

Ex ② given: AB as a part of electric Circuit.
Find H at (0,0,0) due to AB?



Solution

$$\bar{H}_0 = \frac{I}{4\pi r}\left(\sin\alpha_2 - \sin\alpha_1\right)\bar{a}_H$$

$$\alpha_1 = -45^\circ \qquad \alpha_2 = 45^\circ$$

$$r = \sqrt{(1)^2 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2}$$

$$\bar{a}_H = \bar{a}_I \times \bar{a}_r$$

$$\bar{a}_I = ? = \frac{\bar{I}}{|\bar{I}|} = \frac{(0,1,0) - (1,0,0)}{\sqrt{2}} = \frac{-\bar{a}_x + \bar{a}_y}{\sqrt{2}}$$

$$\bar{a}_r = ? = \frac{\bar{r}}{|\bar{r}|} = \frac{(1/2, 1/2, 0) - (0,0,0)}{\sqrt{(1/2)^2 + (1/2)^2}} = \frac{1/\cancel{2}(\bar{a}_x + \bar{a}_y)}{\sqrt{2}/\cancel{2}}$$

$$\therefore \bar{a}_H = \frac{1}{\sqrt{2}}\begin{vmatrix} \bar{a}_x & \bar{a}_y & a_z \\ -1 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \bar{a}_z$$

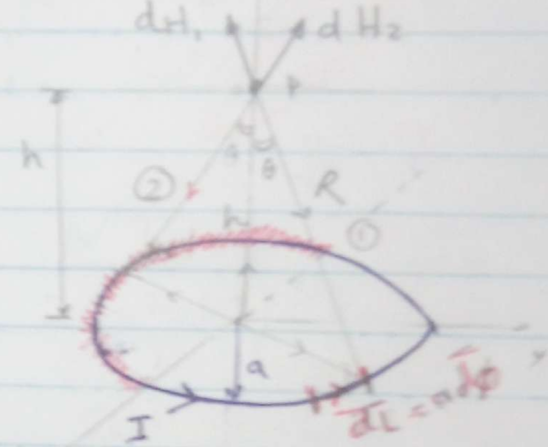
$$\therefore \bar{H}_0 = \frac{6}{4\pi\left(\frac{\sqrt{2}}{2}\right)}\left(\sin 45^\circ - \sin(-45)\right)\bar{a}_z$$

$$= 0.955\ \bar{a}_z\ \text{A/m}$$

5

Sheet 6

1) Required: The magnetic field intensity H on the axis of a circular ring due to current I.



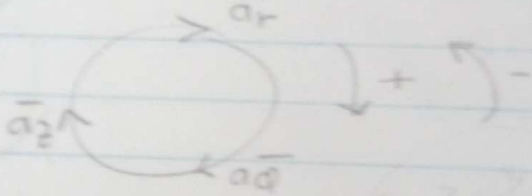
$$d\bar{H} = \frac{I\,d\bar{L} \times \bar{R}}{4\pi R^3}$$

For ① نص الحلقة الأول $0 \to \pi$

$$\bar{dL} = a\,d\phi\,\bar{a}_\phi$$

$$\bar{R} = -a\,\bar{a}_r + h\,\bar{a}_z$$

$$R = \sqrt{h^2 + a^2}$$



$$\bar{dL} \times \bar{R} = -(-a^2\,d\phi\,\bar{a_z}) \boxed{+} a h\,\bar{a}_r$$

For ② as the same نص الحلقة تاني $\pi \to 2\pi$

$$\bar{dL} \times \bar{R} = a^2\,d\phi\,\bar{a_z} \boxed{-} a h\,\bar{a_r}$$

$$dH_1 = \frac{I}{4\pi R^3}\left(a^2\,d\phi\,\bar{a}_z + \cancel{a h\,\bar{a}_r}\right)$$

$$dH_2 = \frac{I}{4\pi R^3}\left(a^2\,d\phi\,\bar{a}_z - a h\,\bar{a_r}\right)$$

$$\bar{dH}_T = \frac{I}{4\pi (a^2+h^2)^{3/2}}\left(2\,a^2\,d\phi\,\bar{a}_z\right)$$

sheet $\boxed{6}$

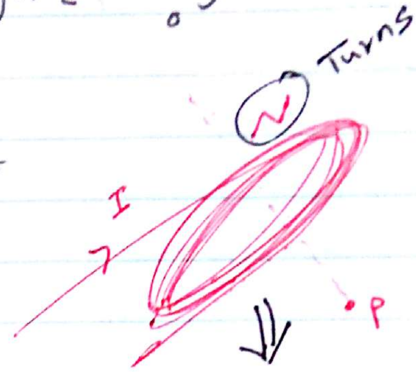
$\boxed{1}$ required: magnetic field intensity H on the axis of a circular loop (current in xy)

Scanned by CamScanner

$\boxed{6}$ ~~3~~

$$\therefore H = \int_{\phi=0}^{\pi} dH = \frac{I\,(2a^2)}{4\pi(a^2+h^2)^{3/2}} \int_0^{\pi} d\phi \;\bar{a}_z$$

$$= \frac{I\,2a^2\,\cancel{\pi}}{{}_2\cancel{4\pi}(a^2+h^2)^{3/2}}\,\bar{a}_z$$



$$H = \frac{I\,a^2}{2(a^2+h^2)^{3/2}}\,\bar{a}_z$$

for N - Turns $\therefore$ $H = \dfrac{N I a^2}{2(a^2+h^2)^{3/2}}\,\bar{a}_z$

if $h=0$ $\Rightarrow$ H عند مركز الحلقة

$$\therefore \bar{H} = \frac{I a^2}{2(a^2)^{3/2}}\,\bar{a}_z$$

$$\frac{I}{2a}\,\bar{a}_z \rightarrow \text{for N-Turns} \quad \bar{H} = \frac{NI}{2a}\,\bar{a}_z$$

(صياغة معادلة H بدلالة الزاوية θ.

Note:

$$H = \frac{N I a^2}{2(a^2+h^2)^{3/2}} \rightarrow = \frac{N I a^2}{2R^3} \times \frac{a}{a}$$

$$= \frac{NI}{2a}\,\frac{a^3}{R^3} \rightarrow \sin\theta = \frac{a}{R}$$

$$H = \frac{NI}{2a}\,\sin^3\theta \quad \rightarrow \quad \text{H بدلالة } \theta$$

for any case To find B $\Rightarrow$ $B = \mu H$ Tesla.

نحدث صياغة للمعادلات على التيار الكلي I وهو الكمية المعروفة

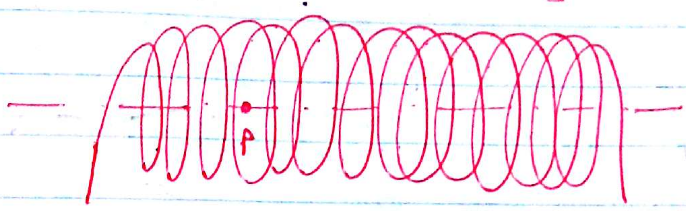
$$J = \frac{NI}{L}$$

كثافة التيار الطولية

I = التيار خلال اللفة الواحدة

التيار الكلي أي في الملف

$$J = \frac{1 * I_{dz}}{dz}$$



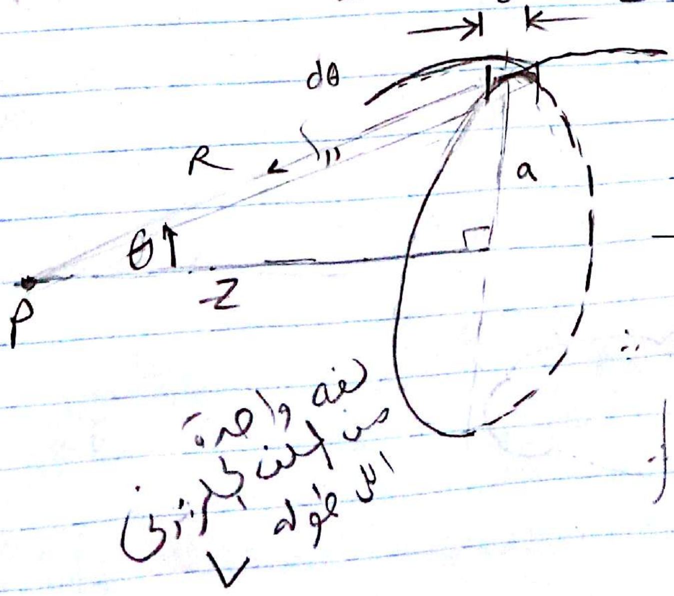
$$\therefore \frac{NI}{L} = \frac{I_{dz}}{dz}$$

$$\therefore \boxed{I_{dz} = \frac{NI}{L} dz}$$

$$\therefore dH_z = \frac{I_{dz}}{2a} \sin\theta^3$$

$$\therefore = \frac{N}{L} \frac{I}{2a} \sin\theta^3 \, dz$$

$$dH_z = \frac{nI}{2a} \sin\theta^3 \, dz$$



$$\sin\theta = \frac{a}{R}$$

$$\tan\theta = \frac{a}{Z}$$

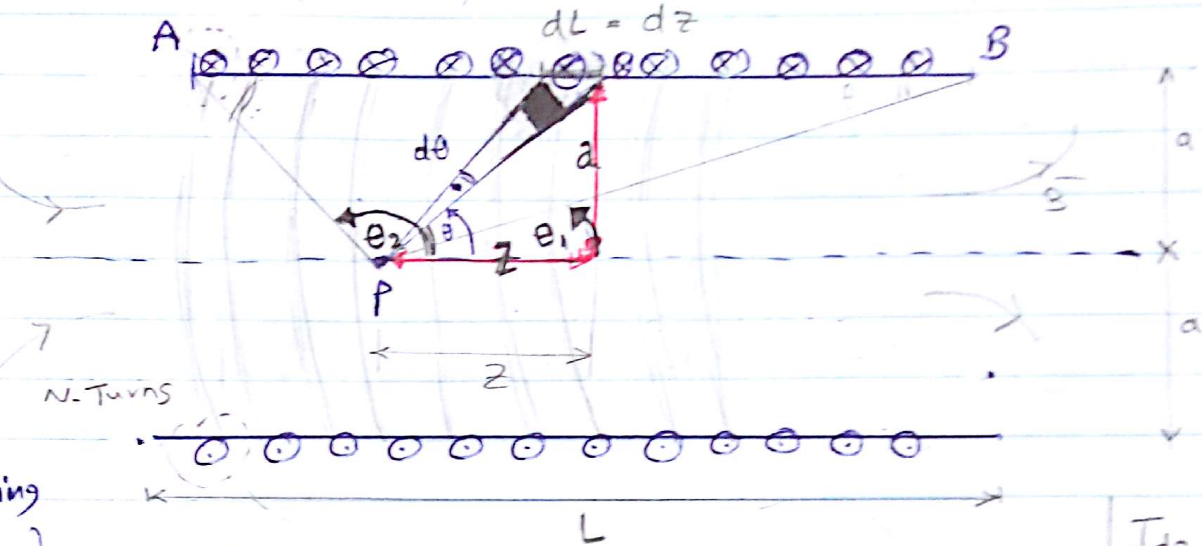
$$\therefore \cot\theta = \frac{Z}{a}$$

$$\boxed{Z = a \cot\theta}$$

(7-1) ~~(4)~~

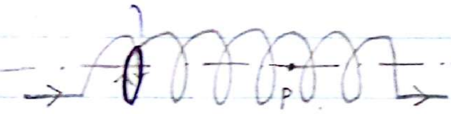
4

$\boxed{2}$ Required: $\bar{H}$ on the axis of Solenoid



$I_{dz} = J \cdot dz$
$= \frac{I}{L} dz$

I: التيار الكلي
J: كثافة التيار الطولية وهي منتظمة
I_{dz}: التيار المار خلال عنصر طولي dz



$\boxed{\text{من اتباع ال Ring:}}$

H بتاع الحلقة ← عشان dH' الحلزوني

$$\overline{dH_z} = \frac{I_{dz}}{2a} \sin\theta^3$$

$$\therefore dH_z = \frac{NI}{2a} \sin\theta^3 \left(\frac{dL}{L}\right) \quad , dL = dz$$

$$= \frac{N}{L}\frac{I}{2a} \sin\theta^3 \; dz$$

عدد اللفات لوحدة الاطوال $\frac{N}{L} =$

$$= \frac{nI}{2a} \sin\theta^3 \; dz$$

$\tan\theta = \frac{a}{z}$

$\therefore \cot\theta = z/a$

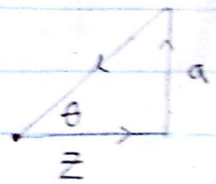
$\therefore z = a\cot\theta$

$dz = -a\,\text{Csec}^2\theta \, d\theta$

$\text{Csec}\,\theta = \frac{1}{\sin\theta}$

$$\therefore \overline{dH_z} = \frac{nI}{2\cancel{a}} \sin\theta^3 \times -\cancel{a}\,\text{Csec}^2\theta \, d\theta \quad \left(\frac{1}{\sin\theta^2}\right)$$

$$= \frac{nI}{2}\left(-\sin\theta\right) d\theta$$



$$\therefore \overline{H_z} = \int_{\theta_1}^{\theta_2} dH_z$$

$$\overline{H_z} = -\frac{nI}{2}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

(19)

8

$$\bar{H}_z = \frac{nI}{2}\left[\cos\theta_1 - \cos\theta_2\right]$$

NOTE:

when P at terminal A $\Rightarrow \theta_2 = \pi/2$

$$\therefore H_z = \frac{nI}{2}\cos\theta_1$$

When P at terminal B $\Rightarrow \theta_1 = \pi/2$

$$H_z = \frac{nI}{2}\cos\theta_2$$

when $\equiv$ infinit solenoid $\theta_1 = 0,\ \theta_2 = \pi$

$$\therefore H = nI$$

$$B = \mu_0 H$$

α_1 : الزاوية بين العمودي وبداية الخط (Ray 1 & Ray 0)

α_2 // // نهاية // (Ray2 & //)

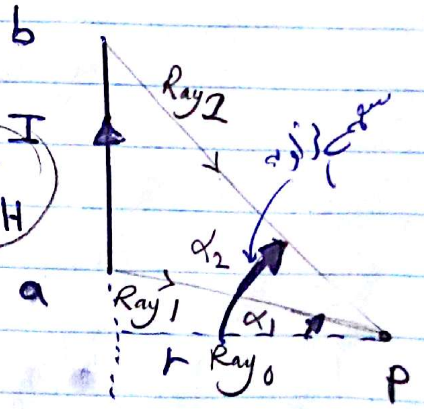
For finit line

$$\bar{a}_H = \bar{a}_I \times \bar{a}_r$$

$$\bar{H}_p = \frac{I}{4\pi r}\left(\sin\alpha_2 - \sin\alpha_1\right)\bar{a}_H$$

للنهاية — للبداية — من الإشارات

البعد العمودي من النقطة للخط



سهم الزاوية مع الساعة ← الزاوية (+) حيث انه سهم الزاوية
عكس ← الزاوية (−) من العمودي للوتر

#

For infinit Line: $\alpha_1 = -90 \quad \alpha_2 = 90$

$$H_p = \frac{I}{2\pi r}\bar{a}_r$$

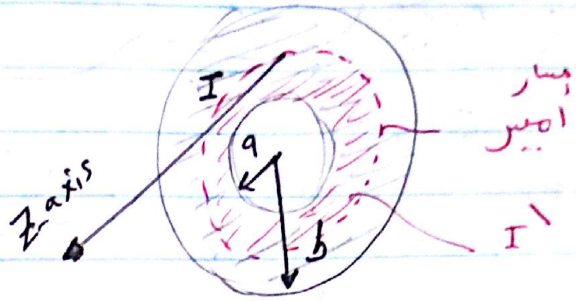
7.17 (Pg.305, S.P182)

14

X

Ex $\boxed{8}$: A hollow Conducting Cylinder, Find $\bar{H}$ everywhere?

Solution



By using Ampere's law

$$\oint \bar{H} \cdot \bar{dl} = I_{enc}$$

For $\rho < a$ $\quad \oint H \cdot dl = \text{Zero} \quad \therefore \bar{H} = \text{Zero}$

$a < \rho < b$ $\quad H_\varphi \int_0^{2\pi} \rho \, d\varphi = I` \Rightarrow$ جزء من التيار الكلي

$$J = \frac{I}{4\pi(b^2 - a^2)}$$ كثافة التيار منتظمة

$$H(2\pi\rho) = I\left(\frac{\rho^2 - a^2}{b^2 - a^2}\right)$$

$$\therefore I` = J \, 4\pi(\rho^2 - b^2)$$

$$= I \frac{\cancel{4\pi}(\rho^2 - a^2)}{\cancel{4\pi}(b^2 - a^2)}$$

$$\therefore \bar{H} = \frac{I}{2\pi\rho}\left(\frac{\rho^2 - a^2}{b^2 - a^2}\right) \bar{a}_\varphi$$

For $\rho > b$ $\quad \oint H \cdot dl = I_{tot}$

$$H(2\pi\rho) = I$$

$$\bar{H}_\varphi = \frac{I}{2\pi\rho} \bar{a}_\varphi$$

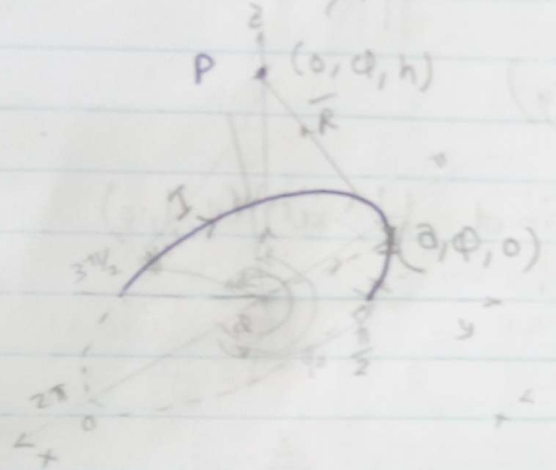
$$\therefore \bar{H}_\varphi = \begin{cases} \text{Zero} & \rho < a \\ \dfrac{I(\rho^2 - a^2)}{(b^2 - a^2)} & a < \rho < b \\ \dfrac{I}{2\pi\rho} & \rho > b \end{cases}$$

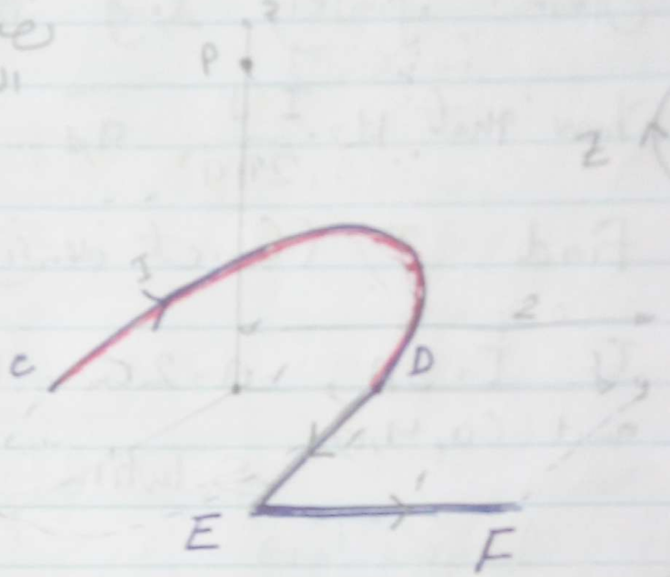
12

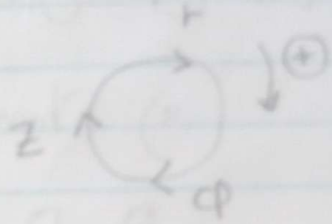
9 ~~6~~

5

$\boxed{3}$ نصف حلقة ← تشكل الشكل لقانون ← نستخدم القانون العام







$R = \sqrt{a^2 + h^2}$

$$\overline{dH} = \frac{I\,\overline{dL} \times \overline{R}}{4\pi\, R^3} \qquad \overline{R} = -a\,\overline{a_r} + h\,\overline{a_z}$$

at Point P on the z-axis

$$\overline{dL} = a\,d\phi\,\overline{a_\phi}$$

$$\therefore \overline{dL} \times \overline{R} = (a\,d\phi\,\overline{a_\phi}) \times (-a\,\overline{a_r} + h\,\overline{a_z})$$

$$= a^2\,d\phi\,\overline{a_z} + ah\,d\phi\,\overline{a_r}$$

$$\therefore \overline{H} = \int_{\phi=\frac{3\pi}{2}}^{\phi=\pi/2} dH = \frac{I a^2}{4\pi R^3}\,\phi\Big|_{\frac{3}{2}\pi}^{\frac{\pi}{2}}\,\overline{a_z} + \frac{I a h}{4\pi R^3}\,\phi\Big|_{\frac{3}{2}\pi}^{\frac{\pi}{2}}\,\overline{a_r}$$

$$\overline{H} = -\frac{I a^2\,\overline{a_z} + I a h\,\overline{a_r}}{4R^3}$$

* نستنتج مركبات $\overline{a_r}$ ، $\overline{a_z}$ وذلك لعدم وجود تماثل للحلقة حول نقطة الاصل . (نصف حلقة فقط)

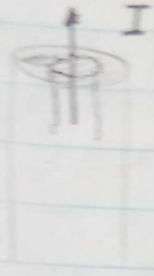
: 1.18
. P 306, S.P 152

(2)

Ex $\boxed{6}$: given: ifinitely long Solid Conductor

a) Show that $H = \frac{I \rho}{2\pi a^2} \bar{a}_\phi$

I (+Z-axis)



b) Find J (Current density)

c) If $I = 3A$, $a = 2cm$. Find H at $(0,1,0)$ and $(0,4,0)$

$\rho = \sqrt{0^2+4^2} = 4$

$\rho = \sqrt{0^2+1^2}$, $\rho = 1$

Solution

a) Using Ampere's law

$$\oint H \cdot dl = I_{enc}$$

$$H_\phi \int_0^{2\pi} \rho \, d\phi = I'$$

$$= I \frac{\rho^2}{a^2}$$

$J = \frac{I}{4\pi a^2}$

$I' = J (4\pi \rho^2)$

$I' = I \frac{\rho^2}{a^2}$

$$H (2\pi \cancel{\rho}) = I \frac{\rho^2}{a^2}$$

$$\therefore \bar{H} = \frac{I \rho}{2\pi a^2} \bar{a}_\phi \quad \#$$

b) $$\bar{J} = \bar{\nabla} \times \bar{H} = -\frac{\partial H_\phi}{\partial z} \bar{a}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho H_\phi) \bar{a}_z$$

ثابت

$$= \frac{I}{\cancel{\rho}} \cdot \frac{1}{2\pi a^2} \cdot 2\cancel{\rho} \, \bar{a}_z = \frac{I}{\pi a^2} \bar{a}_z$$

c) at $(0,1,0) \Rightarrow$ inside the Conductor $\Rightarrow \rho < a$

$$\therefore H_\phi = \frac{I \rho}{2\pi a^2} = \frac{3 \times 1 \times 10^{-2}}{2\pi (4 \times 10^{-4})} = 11.94 \, \bar{a}_\phi \text{ A/m}$$

at $(0,4,0) \Rightarrow$ outside $\Rightarrow \rho > a$

$$\therefore \bar{H}_\phi = \frac{I}{2\pi \rho} \bar{a}_\phi = \frac{3}{2\pi \times 4 \times 10^{-2}} = 11.94 \, \bar{a}_\phi \text{ A/m}$$

[10]

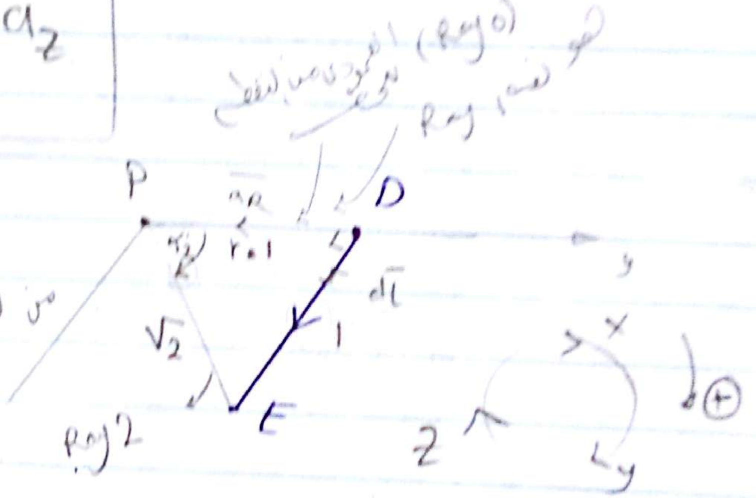
at Center of the half ring $\Rightarrow \therefore h = 0 \quad \therefore \bar{H} = \dfrac{-Ia^2}{4(a^2+0)^{3/2}} \bar{a}_z$

$$\therefore \bar{H} = \frac{-I}{4a} \bar{a}_z$$

$\therefore$ for Part (CD) when $a \to 1$

$$\therefore \boxed{\bar{H}_{P_1} = \frac{-I}{4} \bar{a}_z}$$

for Part (DE)



Finit line $\rightarrow \bar{H}$ من نهاية لنهاية

$\alpha_1 = 0$

$\sin \alpha_2 = \dfrac{1}{\sqrt{2}}$

$\alpha_2 \ (+ve) \rightarrow$ مع اتجاه التيار لأنه اتجاه عكس الزاوية α_2

$\bar{a}_H \rightarrow \bar{a}_I \times (\bar{a}_R)$ اتجاه العمود من السلك للنقطة

$\bar{H}$ لم اتجاه $= \bar{a}_x \times -\bar{a}_y$

$= -\bar{a}_z$

$$\therefore H_P = \frac{I}{4\pi r} (\sin \alpha_2 - \cancel{\sin \alpha_1}) (-\bar{a}_z)$$

$$= \frac{I}{4\pi \times 1} \times \frac{1}{\sqrt{2}} (-\bar{a}_z)$$

$$\boxed{\bar{H}_{P_2} = \frac{-I}{4\sqrt{2}\,\pi} \bar{a}_z}$$

4.20 (Pg. 306, Pg 184)

13

Ex (7) given $\bar{H} = yz(x^2+y^2)\bar{a}_x - y^2 xz\, a_y + 4x^2y^2\bar{a}_z$ A/m

a) Determine J at (5,2,-3)

b) I passing through $x=-1$, $0<y,z<2$

c) Show that $\nabla \cdot B$ = Zero

Solution

a) $J = \nabla \times \bar{H} = \begin{vmatrix} a_x & \bar{a}_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz(x^2+y^2) & -y^2xz & 4x^2y^2 \end{vmatrix}$

$J = (8x^2y + xy^2)\bar{a}_x + \left[y(x^2+y^2) - 4xy^2\right]a_y + \left[-y^2z - z(x^2+y^2)\right]a_z$

at $(5,2,-3) \Rightarrow J = 420\,\bar{a}_x - 22\,\bar{a}_y + 99\,\bar{a}_z$ A/m^2

b) $I = \int \bar{J}\cdot ds_x = \int^2 \int^2 (8x^2y + xy^2)\Big|_{x=-1} dy\,dz$

$ds_x \Leftarrow \bar{ds}$: الباقي) x=-1 لان z=0 y=0

$\bar{J}_x \leftarrow \bar{J}$ لان : $= 53.33$ A

c) $\bar{B} = \mu \bar{H}$, $\nabla \cdot \bar{B} = 0 \rightarrow \bar{\nabla} \cdot \bar{H} = 0$

لو اثبتنا ان $\nabla \cdot H$ = صفر ← ∴ $\nabla \cdot B = 0$

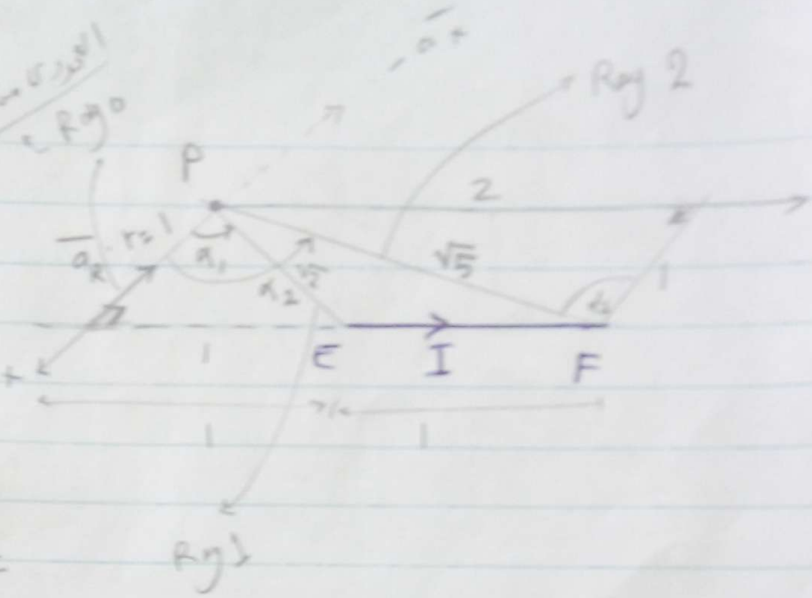
∵ $\nabla \cdot H = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z}$

$= 2xyz - 2xyz$ = Zero

Hence, ∴ $\nabla \cdot \bar{B} = 0$ #

~~8~~ 11

For (EF)



α_1 & α_2 (+ve)

$\sin \alpha_1 = \frac{1}{\sqrt{2}}$

$\sin \alpha_2 = \frac{2}{\sqrt{5}}$

$a_H = a_I \times a_R = \bar{a}_y \times -\bar{a}_x = \bar{a}_z$

$\therefore \bar{H}_{P_3} = \frac{I}{4\pi r} (\sin \alpha_2 - \sin \alpha_1)\, \bar{a}_H$

$$\boxed{H_{P_3} = \frac{I}{4\pi \times 1} \left(\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{2}} \right) \bar{a}_z}$$

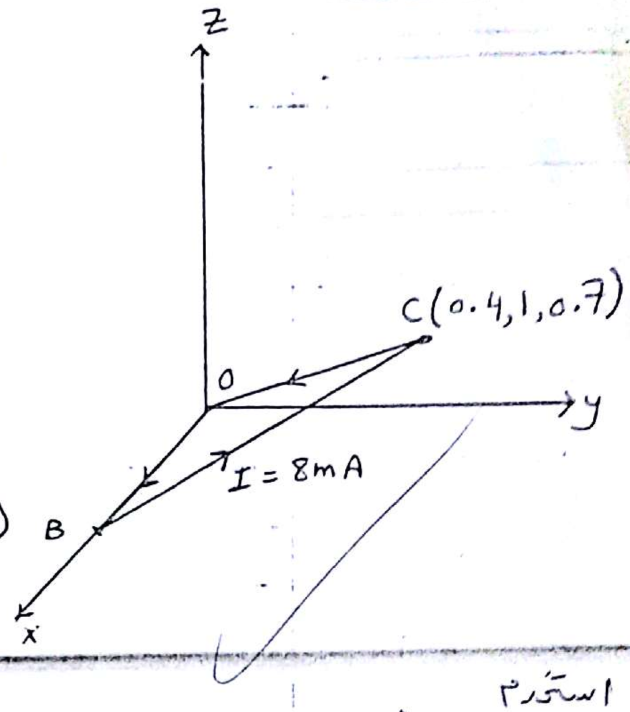
$\therefore \bar{H}_{TOT}$ at $P \Rightarrow \bar{H}_P = \bar{H}_{P_1} + \bar{H}_{P_2} + \bar{H}_{P_3}$

$$\bar{H}_P = \frac{-I}{4} \bar{a}_z - \frac{I}{4\sqrt{2}\pi} \bar{a}_z + \frac{I}{4\pi} \left(\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{2}} \right) \bar{a}_z$$

Force and Torque

Problem (1)

$$B = 0.2\,a_x - 0.1\,a_y + 0.2\,a_z$$



Find force on OBC

Solution

$$F = F_{OB} + F_{BC} + F_{CO}$$

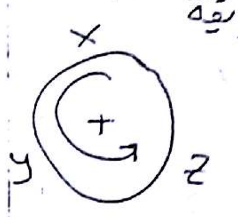
F_{OB} $\quad F_{OB} = I \int d\bar{L} \times \bar{B}$

$$d\bar{L} = dx\, a_x$$

استخدم المحدد أو أي طريقة

$$\therefore d\bar{L} \times \bar{B} = dx\, \bar{a}_x \times (0.2\, a_x - 0.1\, a_y + 0.2\, a_z)$$

$$= -0.1\, dx\, \bar{a}_z + 0.2\, dx\, (-a_y)$$



$$= -0.1\, dx\, a_z - 0.2\, dx\, a_y$$

$$F_{OB} = I \int_0^{0.6} (-0.1\, dx\, a_z - 0.2\, dx\, a_y)$$

$$= 8 \times 10^{-3} \left[-0.1 \times 0.6\, a_z - 0.2 \times 0.6\, a_y\right]$$

$$F_{OB} = (-4.8\, a_z - 9.6\, a_y) \times 10^{-4} \text{ N.}$$

F_{BC} $\quad F_{BC} = I \int d\bar{L} \times \bar{B} \qquad d\bar{L} = dx\, a_x + dy\, a_y + dz\, a_z$

$$dL \times B = \begin{vmatrix} \overset{+}{a_x} & \overset{-}{a_y} & \overset{+}{a_z} \\ dx & dy & dz \\ 0.2 & -0.1 & 0.2 \end{vmatrix} = (0.2\,dy + 0.1\,dz)\, a_x - (0.2\,dx - 0.2\,dz)\, a_y + (-0.1\,dx - 0.2\,dy)\, a_z$$

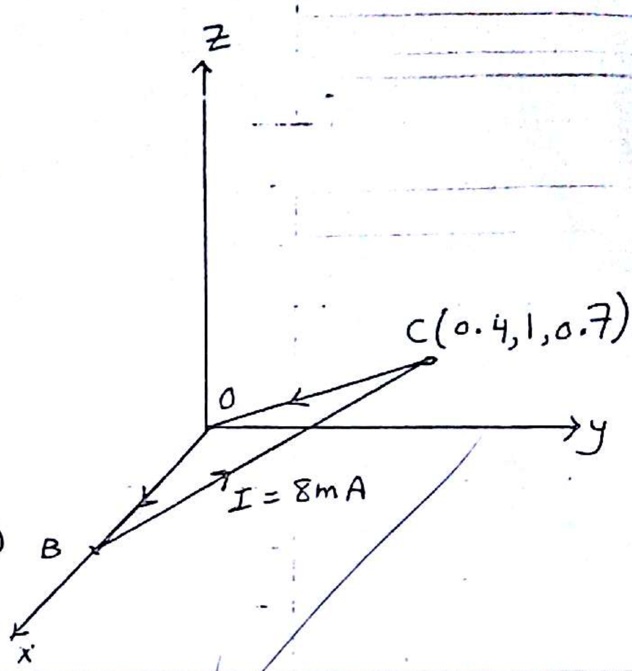
$$F_{BC} = I \int_{x=0.6}^{0.4} \int_{y=0}^{1} \int_{z=0}^{0.7} \text{---}$$

Force and Torque

Problem (1)

$$B = 0.2\,a_x - 0.1\,a_y + 0.2\,a_z$$

find force on OBC



Solution

$$F = F_{OB} + F_{BC} + F_{CO}$$

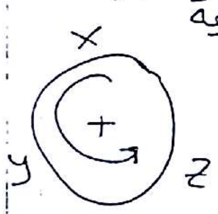
F_{OB} $\qquad F_{OB} = I \int d\bar{L} \times \bar{B}$

$$d\bar{L} = dx\,a_x$$

$$\therefore d\bar{L} \times \bar{B} = dx\,\bar{a}_x \times (0.2\,a_x - 0.1\,a_y + 0.2\,a_z)$$

استخدم
المحدد أو
أي طريقة

$$= -0.1\,dx\,\bar{a}_z + 0.2\,dx\,(-a_y)$$



$$= -0.1\,dx\,a_z - 0.2\,dx\,a_y$$

$$F_{OB} = I \int_0^{0.6} (-0.1\,dx\,a_z - 0.2\,dx\,a_y)$$

$$= 8 \times 10^{-3} \left[-0.1 \times 0.6\,a_z - 0.2 \times 0.6\,a_y \right]$$

$$F_{OB} = (-4.8\,a_z - 9.6\,a_y) \times 10^{-4} \quad N.$$

F_{BC} $\qquad F_{BC} = I \int d\bar{L} \times \bar{B} \qquad d\bar{L} = dx\,a_x + dy\,a_y + dz\,a_z$

$$dL \times B = \begin{vmatrix} \overset{\oplus}{a_x} & \overset{\ominus}{a_y} & \overset{\oplus}{a_z} \\ dx & dy & dz \\ 0.2 & -0.1 & 0.2 \end{vmatrix} = (0.2dy + 0.1dz)\,a_x - (0.2dx - 0.2dz)\,a_y + (-0.1dx - 0.2dy)\,a_z$$

$$F_{BC} = I \int_{x=0.6}^{0.4} \int_{y=0}^{1} \int_{z=0}^{0.7} \cdots$$

$$F_{BC} = 8*10^{-3}\Big[(0.2*1 + 0.1*0.7)\,a_x$$
$$- (0.2*(0.4-0.6) - 0.2*0.7)\,a_y$$
$$+ (-0.1*(0.4-0.6) - 0.2*1)\,a_z$$

$$= (2.16\,a_x + 1.44\,a_y - 1.44\,a_z)*10^{-3}\ N.$$

$\underline{F_{CO}}$ $\quad dL = dX\,a_x + dy\,a_y + dZ\,a_z$ حدود التكامل متغيرة الاتجاه.

$\bar{dL} \times \bar{B} \rightarrow$ نفس الطريقة السابقة

$$F_{CO} = 8*10^{-3}\int_{X=0.4}^{0}\int_{y=1}^{0}\int_{Z=0.7}^{0} ----$$

$$= 8*10^{-3}\Big[(0.2*-1 + 0.1*-0.7)\,a_x$$
$$- (0.2*-0.4 - 0.2*-0.7)\,a_y$$
$$+ (-0.1*-0.4 - 0.2*-1)\,a_z\Big]$$

$$F_{CO} = (-2.16\,a_x - 0.48\,a_y + 1.92\,a_z)*10^{-3}\ N$$

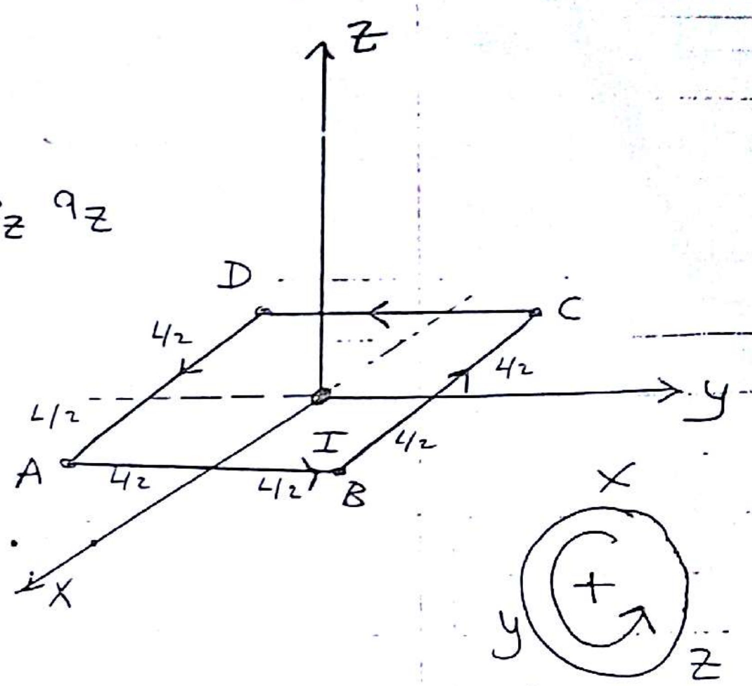
$$F_t = F_{OB} + F_{BC} + F_{CO}\ N$$

$$F_t = \text{Zero}\ N \quad \#$$

Problem (2):

$$B = B_x a_x - B_y a_y + B_z a_z$$

Find F



Solution

$$F = I \int dL \times B$$

وهنا 4 أجزاء

AB

$$dL = dy \, \bar{a}_y$$

$$dL \times B = dy \, \bar{a}_y \times (B_x a_x - B_y a_y + B_z a_z)$$

$$= B_x dy \cdot (-a_z) + B_z dy \, (a_x)$$

أو استخدم الجدرات

$$dL \times B = dy \, (-B_x a_z + B_z a_x)$$

$$F_{AB} = I \int_{y=-L/2}^{L/2} dL \times B = I\left[-B_x a_z + B_z a_x\right] L$$

$$= LI\,(-B_x a_z + B_z a_x)$$

BC

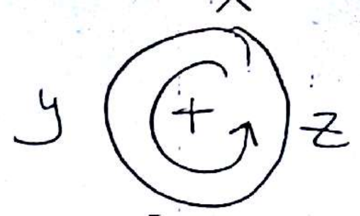
$$dL = dx \, a_x$$

$$dL \times B = dx \, a_x \times (B_x a_x - B_y a_y + B_z a_z)$$

$$= dx\,(-B_y a_z + B_z(-a_y))$$

$$F_{BC} = I \int_{x=+L/2}^{-L/2} dL \times B = -LI\left[-B_y a_z - B_z a_y\right]$$

$$= LI\left[B_y a_z + B_z a_y\right]$$



CD $\quad dL = dy \; a_y$

$$dL \times \bar{B} = dy \; a_y \times (B_x a_x - B_y a_y + B_z a_z)$$

$$= dy(-B_x a_z + B_z a_x)$$

$$F_{CD} = I\int_{y=+L/2}^{-L/2} dL \times B = -IL(-B_x a_z + B_z a_x)$$

$$= IL(B_x a_z - B_z a_x)$$

DA $\quad dL = dx \; a_x$

$$dL \times B = dx \; a_x (B_x a_x - B_y a_y + B_z a_z)$$

$$= dx(-B_y a_z + B_z(-a_y))$$

$$F_{DA} = I\int_{x=-L/2}^{L/2} \bar{dL} \times B = IL[-B_y a_z - B_z a_y]$$

$$\therefore F_t = F_{AB} + F_{BC} + F_{CD} + F_{DA} = \text{Zero} \;\#$$

Conclusion:-

In uniform field $\rightarrow$ Force on closed path

$$F = I\oint dL \times B = -IB \times \oint dL$$

$$= \text{Zero}$$

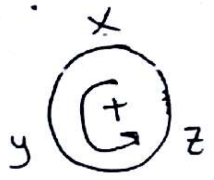
$$\oint dL = \text{Zero}$$

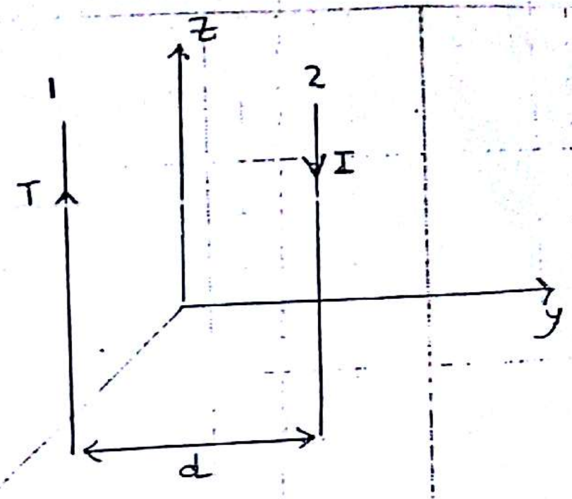
القوة المؤثرة على مسار مغلق في مجال منتظم تساوي صفر

3. Force between two parallel lines.

Currents in Opposite directions

Field on ① due to ②





$$H_{21} = \frac{I}{2\pi d}\, a_\phi$$

$$a_\phi = a_{I_2} \times a_R = -a_z \times (-a_y)$$

المركبة المحورية

$$= -a_x$$

$$H_{21} = \frac{I}{2\pi d}(-a_x)$$

$$B_{21} = \frac{\mu_0 I}{2\pi d}(-a_x)$$

$$dL_1 = dz\, a_z$$

$$dL_1 \times B_{21} = dz\, a_z \times \left(\frac{\mu_0 I}{2\pi d}\right)(-a_x)$$

$$= -\frac{\mu_0 I}{2\pi d}\, dz\, (a_z \times a_x)$$

$$= \frac{-\mu_0 I}{2\pi d}\, dz\, (a_y)$$

Force on ① $\quad F_1 = I \int d\bar{L}_1 \times \bar{B}_{21}$

$$F_1 = I \int_0^{\text{length}} \frac{-\mu_0 I}{2\pi d}\, dz\; a_y$$

$$F_1 = -\frac{\mu_0 I^2}{2\pi d}(\text{length})\; a_y$$

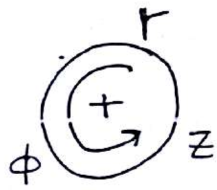
$$\frac{F_1}{\text{length}} = \frac{\mu_0 I^2}{2\pi d}(-a_y) \rightarrow \text{Repulsive force}$$ قوة تنافر

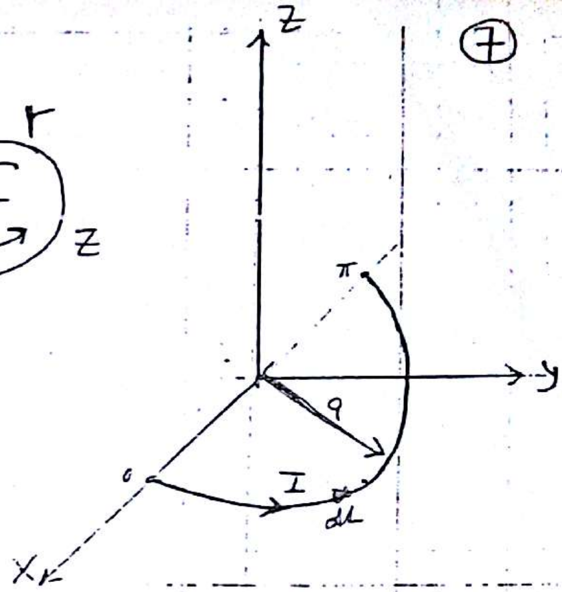
$|F_2| = |F_1|$ دفي الاتجاه العكس

(a_y)

B is on a_z direction

$\boxed{5}$ $\bar{B} = B\, a_z$





<u>Find Force</u>

$$F = I \int dL \times B$$

$$\bar{dL} = a\, d\phi \; a_\phi \qquad \bar{B} = B\, a_z$$

$$\bar{dL} \times \bar{B} = a B\, d\phi \,(a_\phi \times a_z)$$

$$= a B\, d\phi \; \bar{a}_r$$

$$F = I \int_{\phi=0}^{\pi} a B\, d\phi \; \bar{a}_r$$

$$F = \pi a B I \; \bar{a}_r \quad \#$$

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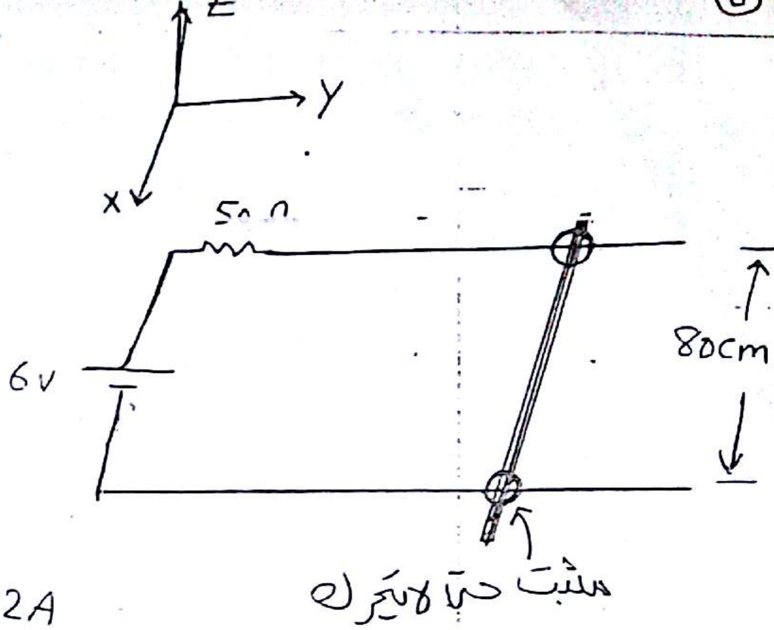
6

$$\bar{B} = 0.5\, a_z$$

احسب القوة المؤثرة على bar



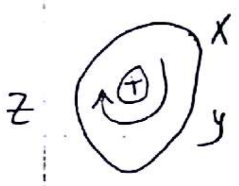
Solution

$$I = \frac{V}{R} = \frac{6}{50} = 0.12A$$

$$\bar{L} = 0.8\, \bar{a}_x$$

$$\bar{F} = I \bar{L} \times \bar{B} = (0.12 * 0.8 * 0.5)(\bar{a}_x \times \bar{a}_z)$$

$$\bar{F} = 0.048\,(-\bar{a}_y)$$



$$|F| = 0.048 \ \ N$$

direction $\bar{a}_F = -\bar{a}_y$ #

As a numerical example of these equations, consider Fig. 9.2. We have a square loop of wire in the $z = 0$ plane carrying 2 mA in the field of an infinite filament on the y axis, as shown. We desire the total force on the loop.

Solution. The field produced in the plane of the loop by the straight filament is

$$\mathbf{H} = \frac{I}{2\pi x}\mathbf{a}_z = \frac{15}{2\pi x}\mathbf{a}_z \quad \text{A/m}$$

Therefore,

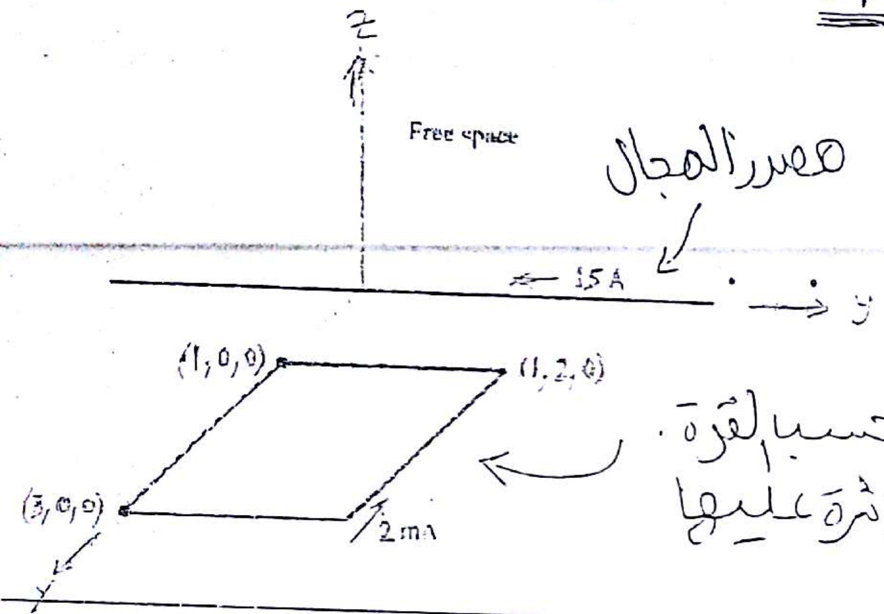
$$\mathbf{B} = \mu_0 \mathbf{H} = 4\pi \times 10^{-7}\mathbf{H} = \frac{3 \times 10^{-6}}{x}\mathbf{a}_z \quad \text{T}$$

We use the integral form (10),

$$\mathbf{F} = -I\oint \mathbf{B} \times d\mathbf{L}$$

or $\bar{F} = I\int d\bar{L} \times \bar{B}$

نفس الحل .



مصدر المجال

احسب القوة المؤثرة عليها

Let us assume a rigid loop so that the total force is the sum of the forces on the four sides. Beginning with the left side:

$$\mathbf{F} = -2 \times 10^{-3} \times 3 \times 10^{-6}\left[\int_{x=1}^{3} \frac{\mathbf{a}_z}{x} \times dx\,\mathbf{a}_x + \int_{y=0}^{2} \frac{\mathbf{a}_z}{3} \times dy\,\mathbf{a}_y + \int_{x=3}^{1} \frac{\mathbf{a}_z}{x} \times dx\,\mathbf{a}_x + \int_{y=2}^{0} \frac{\mathbf{a}_z}{1} \times dy\,\mathbf{a}_y\right]$$

$$= -6 \times 10^{-9}\left[\ln x\Big|_1^3 \mathbf{a}_y + \frac{1}{3}y\Big|_0^2(-\mathbf{a}_x) + \ln x\Big|_3^1 \mathbf{a}_y + y\Big|_2^0(-\mathbf{a}_x)\right]$$

$$= -6 \times 10^{-9}\left[(\ln 3)\mathbf{a}_y - \frac{2}{3}\mathbf{a}_x + \left(\ln\frac{1}{3}\right)\mathbf{a}_y + 2\mathbf{a}_x\right]$$

$$= -8\mathbf{a}_y \quad \text{pN}$$

أو تأخذها كل ضلع لوحده في مرة للتسهيل على نفسك

Thus, the net force on the loop is in the $-\mathbf{a}_y$ direction.

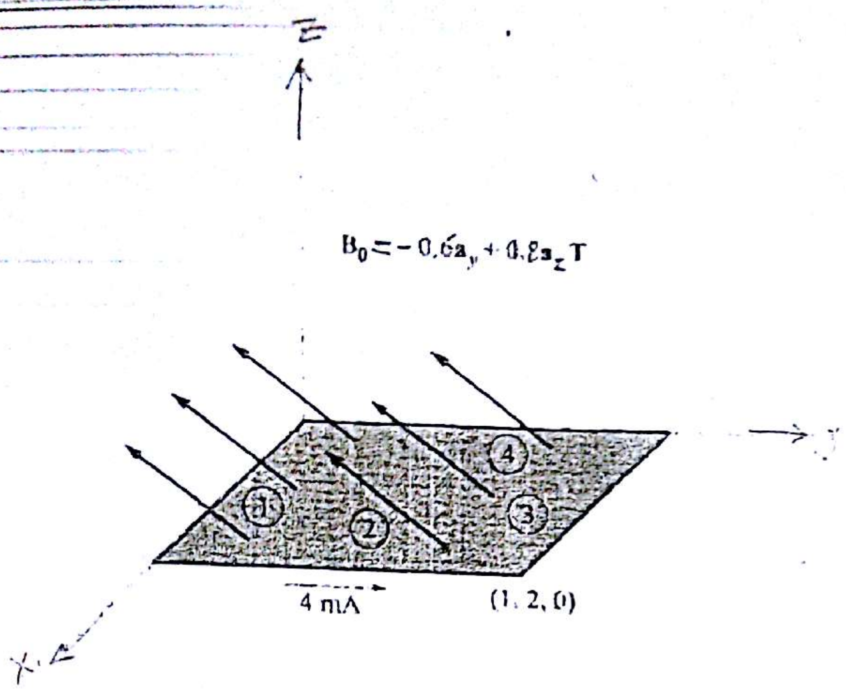


FIGURE 9.7
A rectangular loop is located in a uniform magnetic flux density $\mathbf{B}_0$.

Example 9.3

To illustrate some force and torque calculations, consider the rectangular loop shown in Fig. 9.7. Calculate the torque by using $\mathbf{T} = I\mathbf{S} \times \mathbf{B}$.

Solution. The loop has dimensions of 1 m by 2 m and lies in the uniform field $\mathbf{B}_0 = -0.6\mathbf{a}_y + 0.8\mathbf{a}_z$ T. The loop current is 4 mA, a value that is sufficiently small to avoid causing any magnetic field that might affect $\mathbf{B}_0$.

We have

$$\mathbf{T} = 4 \times 10^{-3}[(1)(2)\mathbf{a}_z] \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z) = 4.8\mathbf{a}_x \quad \text{mN} \cdot \text{m}$$

Thus, the loop tends to rotate about an axis parallel to the positive x axis. The small magnetic field produced by the 4-mA loop current tends to line up with $\mathbf{B}_0$.

Example 9.4

Now let us find the torque once more, this time by calculating the total force and torque contribution for each side.

Solution. On side 1 we have

$$\mathbf{F}_1 = I\mathbf{L}_1 \times \mathbf{B}_0 = 4 \times 10^{-3}(1\mathbf{a}_x) \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z)$$
$$= -3.2\mathbf{a}_y - 2.4\mathbf{a}_z \quad \text{mN}$$

On side 3 we obtain the negative of this result,

$$\mathbf{F}_3 = 3.2\mathbf{a}_y + 2.4\mathbf{a}_z \quad \text{mN}$$

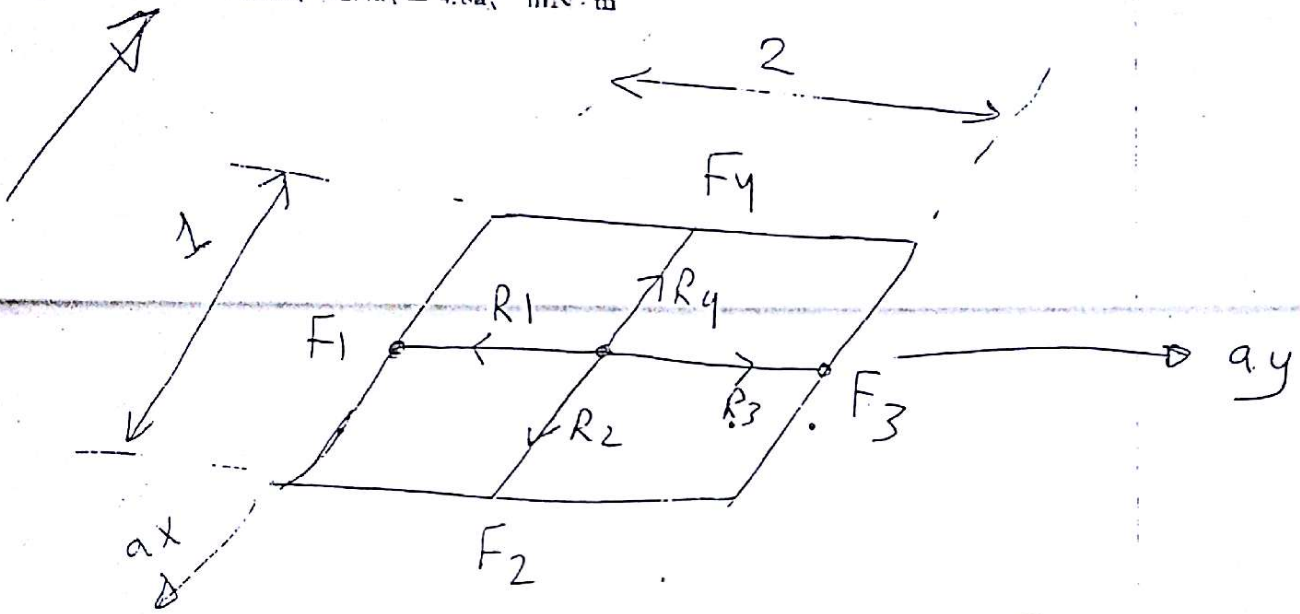
$$\mathbf{F}_2 = I\mathbf{L}_2 \times \mathbf{B}_0 = 4 \times 10^{-3}(2\mathbf{a}_y) \times (-0.6\mathbf{a}_x + 0.8\mathbf{a}_z)$$
$$= 6.4\mathbf{a}_x \quad \text{mN}$$

with side 4 again providing the negative of this result.

$$\mathbf{F}_4 = -6.4\mathbf{a}_x \quad \text{mN}$$

Since these forces are distributed uniformly along each of the sides, we treat each force as if it were applied at the center of the side. The origin for the torque may be established anywhere since the sum of the forces is zero, and we choose the center of the loop. Thus,

$$\mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 + \mathbf{T}_4 = \mathbf{R}_1 \times \mathbf{F}_1 + \mathbf{R}_2 \times \mathbf{F}_2 + \mathbf{R}_3 \times \mathbf{F}_3 + \mathbf{R}_4 \times \mathbf{F}_4$$
$$= (-1\mathbf{a}_y) \times (-3.2\mathbf{a}_y - 2.4\mathbf{a}_z) + (0.5\mathbf{a}_x) \times (6.4\mathbf{a}_x)$$
$$+ (1\mathbf{a}_y) \times (3.2\mathbf{a}_y + 2.4\mathbf{a}_z) + (-0.5\mathbf{a}_x) \times (-6.4\mathbf{a}_x)$$
$$= 2.4\mathbf{a}_x + 2.4\mathbf{a}_x = 4.8\mathbf{a}_x \quad \text{mN}\cdot\text{m}$$



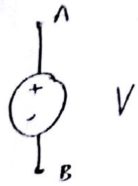
$$\begin{cases} R_1 = -1\,a_y \\ R_3 = 1\,a_y \end{cases}$$

$R_2 = 0.5\,a_x$

$R_4 = -0.5\,a_x$

* Magnetic Circuit

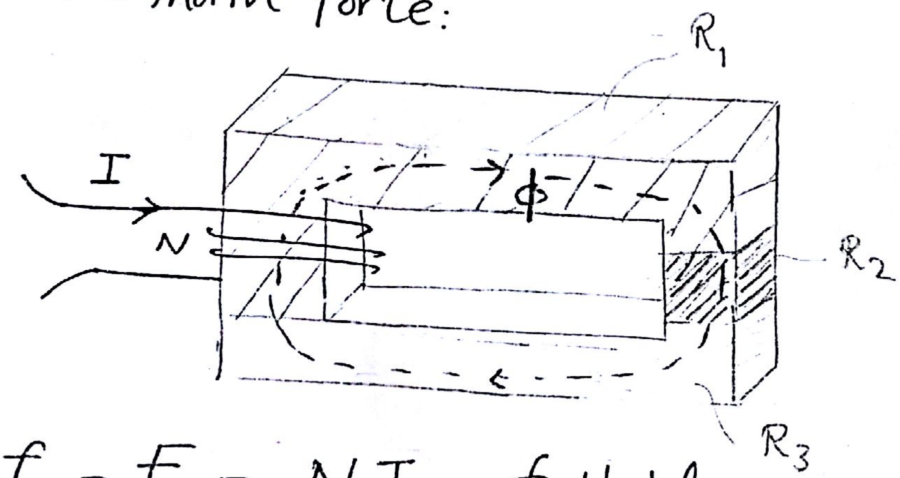
In electric field: $V_{AB} = -\int_A^B E \cdot dl$



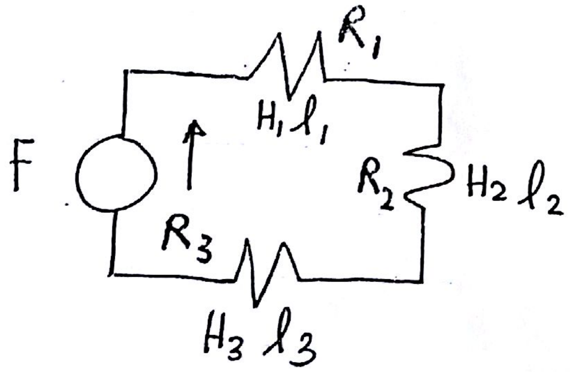
For magnetic field

mmF $= V_{mmf_{AB}} = F = \int_A^B H \cdot dl$

✓ Magnetic motive force:



$$mmf = F = NI = \oint H \cdot dl$$



$$F = \int_1 H_1 \cdot dl + \int_2 H_2 \cdot dl + \int_3 H_3 \cdot dl$$

$$= H_1 L_1 + H_2 l_2 + H_3 l_3$$

$$\therefore NI = \sum HL$$

Magnetic Torque and Moment

$$\bar{m} = I S \, \bar{a}_n \quad A/m^2$$

magnetic dipole moment

$$\bar{T} = \bar{m} \times \bar{B}$$

Magntization in materials

$$B = \mu_o (H + M), \quad M = \chi_m H$$

χ_m: Magntization — magnetic susceptibility of the medium

$$J_b = \bar{\nabla} \times \bar{M}, \quad K_b = \bar{M} \times a_n$$

K_b : bound surface current density

J_b : " volume "

EX (8.7) Pg. 336

Classification of Magnetic materials

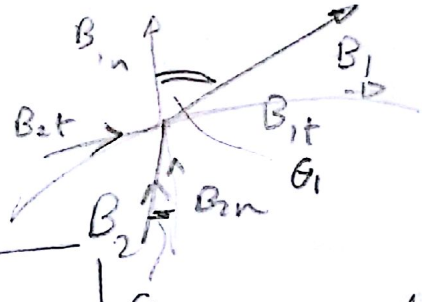
- Diamagnetic $\mu_r < 1.0$
- Para " $\mu_r \geq 1$
- Ferro " $\mu_r \gg 1$

- Magnetic Boundary Conditions

$$\oint B \cdot ds = \text{Zero}, \quad \oint H \cdot dl = I_{enc}$$

$$\boxed{B_{1n} = B_{2n}}$$

$$\Rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$$



$$\boxed{H_{1t} = H_{2t}}$$

$$\boxed{(H_1 - H_2) \times a_{n12} = K}$$

K → free current density

a_{n12} → unit vector normal to the interface and directed from ① → ②

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

for free of current media or Not conductors

$\therefore \boxed{K = 0}$

EX 8.8) Pg. 338

and EX 8.9) Pg. 340

Magnetic Torque and Moment

$$\bar{m} = I S \, \bar{a}_n \qquad A/m^2$$

magnetic dipole moment

$$\bar{T} = \bar{m} \times \bar{B}$$

Magnetization in materials

$$B = \mu_0 (H + M), \qquad M = \chi_m H$$

Magnetization

χ_m: magnetic susceptibility of the medium

$$\bar{J}_b = \bar{\nabla} \times \bar{M}, \qquad K_b = \bar{M} \times \bar{a}_n$$

K_b : bound surface current density

J_b : " volume "

EX 8.7 Pg. 336

classification of Magnetic materials

- Diamagnetic $\mu_r < 1.0$
- Para " $\mu_r \gtrsim 1$
- Ferro " $\mu_r \gg 1$

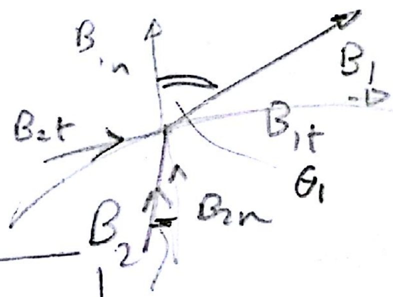
- Magnetic Boundary Conditions

$$\oint B \cdot ds = \text{zero}, \qquad \oint H \cdot dl = J_{enc}$$

~~From~~

$$\boxed{B_{1n} = B_{2n}}$$

$$\Rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$$



$$\boxed{H_{1t} = H_{2t}}$$

$$\boxed{(H_1 - H_2) \times a_{n12} = K}$$

K : free current density

a_{n12} : unit vector normal to the interface and directed from ① → ②

for free of current media or Not conductors

$\therefore \boxed{K = 0}$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

EX 8.8 Pg. 338

and EX 8.9 Pg. 340

$$\because\ H = \frac{B}{\mu} \quad , \quad B = \frac{\phi}{A}$$

طول مسار الفيض

المساحة اللي يخترقها الفيض

$$\therefore NI = \sum \phi \frac{L}{\mu A} \quad , \quad \because R = \frac{L}{\mu A}$$

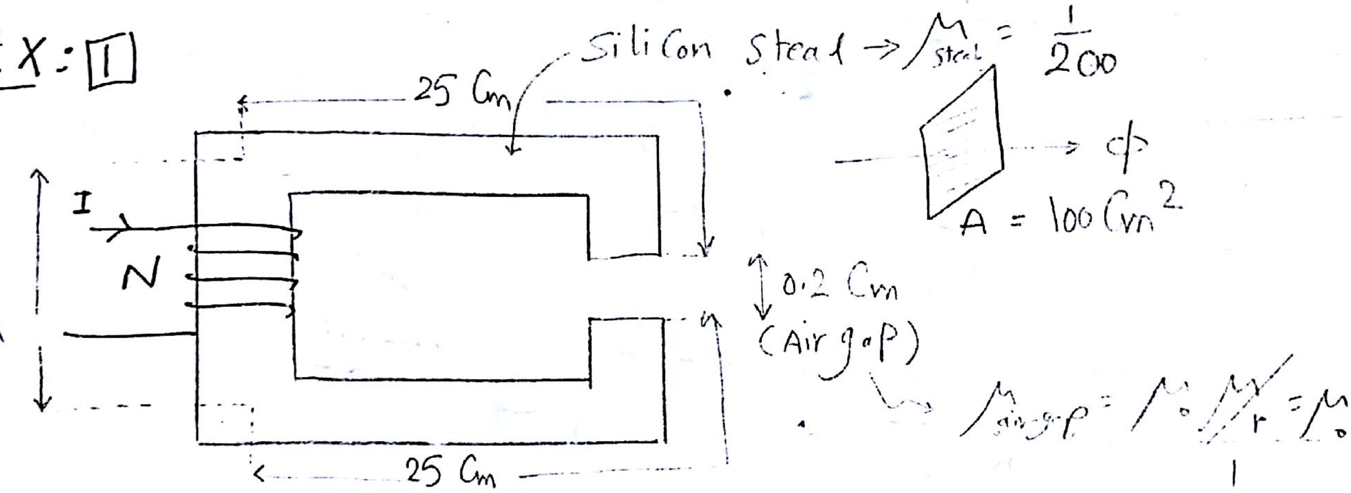
magnetic Reluctance

الممانعة المغناطيسية

$$\therefore NI = \sum \phi R$$

$$NI = \phi (R_1 + R_2 + R_3)$$

EX: $\boxed{1}$



Find N ? ⇒ B = 1 Tesla , I = 10 A

Solution

$mmf_{tot} = NI = mmf_{steel} + mmf_{air}$

(1) $mmf_{air} = \phi R_{air}$

$$\phi = B * A_g = 1 * 25 * 10^{-4} = 25 * 10^{-4}\ wb$$

$$R_{air} = \frac{l_g}{\mu_0 \cancel{\mu_r} A_g} = \frac{0.2 * 10^{-2}}{\mu_0 * 25 * 10^{-4}} \quad , \mu_0 = 4\pi * 10^{-7}$$

$$= 636 * 10^{3}\ AT/wb$$

$$\therefore mmf_{air} = 25 * 10^{-4} * 636.6 * 10^{3} = 1590\ AT$$

EX 8.7 pg 336

given: Region $0 \le Z \le 2$ is occupied by material ($\mu_r = 2.5$). if $B = 10y\,\bar{a}_x - 5x\,\bar{a}_y \ \frac{mWb}{m^2}$

determine: J, J_b, M, K_b on $Z=0$

Solution

a) $J = \nabla \times H = \nabla \times \frac{B}{\mu_0 \mu_r}$

$$= \frac{1}{\mu_0 (2.5)} \left[\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right] \bar{a}_Z$$

$$= \frac{10^6}{\pi} (-5 - 10) \times 10^{-3}\ a_Z$$

$$J = -4.775\ \bar{a}_Z \quad KA/m^2$$

b) J_b (bound volume current density)

$$= X_m J = (\mu_r - 1) J$$

$$= (2.5-1) \times -4.775 \times 10^3$$

$$J_b = -7.163 \quad KA/m^2$$

c) $M = X_m H = X_m \frac{B}{\mu_0 \mu_r} = (\mu_r - 1)\left(\frac{B}{\mu_0 \mu_r}\right)$

(magnetization)

$$= \frac{1.5 \times [10y\ a_x - 5x\ a_y]}{\mu_0 (2.5)} = 4.775y\,\bar{a}_x - 2.387x\,\bar{a}_y \ \frac{KA}{m^2}$$

d) K_b (bound surface current density)

$K_b = M \times a_n \quad \because Z=0 \therefore a_n = -Z_a$

$$= (4.77y\,\bar{a}_x - 2.387x\,\bar{a}_y) \times (-\bar{a}_Z)$$

$$\boxed{K_b = 2.387x\,\bar{a}_x + 4.775y\,\bar{a}_y \quad KA/m}$$

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(2) $mmf_{Steal} = mmf_{25Cm} + mmf_{25Cm} + mmf_{10Cm}$

$$= 2 * \phi R_{25Cm} + \phi * R_{10Cm}$$

$$R_{25Cm} = \frac{l_{25Cm}}{\mu_{steal} A_s} = \frac{25*10^{-2}}{\frac{1}{200}*25*10^{-4}} = \underline{2000\ AT/wb}$$

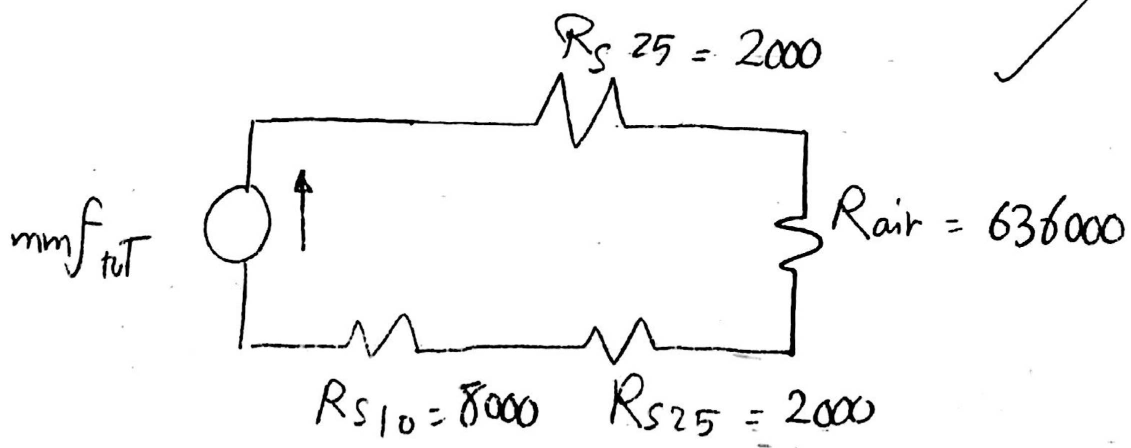
$$R_{10Cm} = \frac{l_{10Cm}}{\mu_{steal} A_s} = \frac{10*10^{-2}}{\frac{1}{200}*25*10^{-4}} = \underline{8000\ AT/wb}$$

$$\therefore mmf_{Steal} = 2 * 25*10^{-4}(2000) + 25*10^{-4}*8000$$
$$= 30\ AT$$

$$\therefore mmf_{tot} = 1590 + 30 = 1620\ \text{Tesla}$$

$$= N I = N * 10$$

$$\therefore N = \frac{1620}{10} = 162\ \text{Turn}$$



EX 8.8 Pg 338

Given: $\bar{H} = -2\bar{a}_x + 6\bar{a}_y + 4\bar{a}_Z$ A/m

in region $y - x - 2 \leq 0$ where

$\mu_1 = 5\mu_0$ c/ Calc:

a) M_1 & B_1

b) H_2 & B_2 in region $y - x - 2 \geq 0$

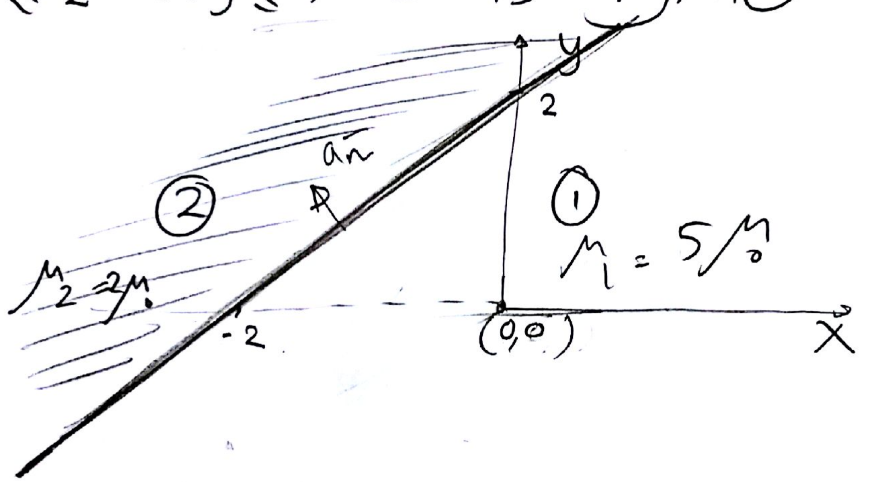
where $\mu_2 = 2\mu_0$

Solution

since

$\therefore$ $y - x - 2 = 0$ is the Plane

$\therefore$ $y - x \leq 2$ or $y \leq x + 2$ is region ①



magnetization

a) $M_1 = \chi_{m_1} H_1$

$= (\mu_{r_1} - 1) H_1 = (5 - 1)(-2, 6, 4)$

$= -8\bar{a}_x + 24\bar{a}_y + 16 a_Z$ A/m

flux density

$B_1 = \mu_1 H_1 = \mu_0 \mu_{r_1} H_1 = \mu_0 (5)(-2, 6, 4)$

$= -12.57\bar{a}_x + 37.7\bar{a}_y + 25.13\bar{a}_Z$ μwb/m²

b) $H_{1n} = (H_1 \cdot a_n) a_n$

$= \left[(-2, 6, 4) \cdot \left(\frac{-1, 1, 0}{\sqrt{2}}\right)\right] \left(\frac{(-1, 1, 0)}{\sqrt{2}}\right)$

$\bar{H}_{1n} = -4\bar{a}_x + 4\bar{a}_y$

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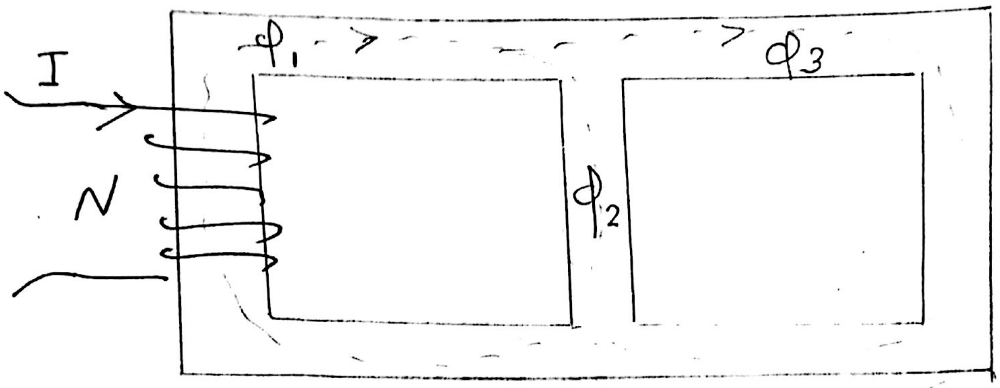
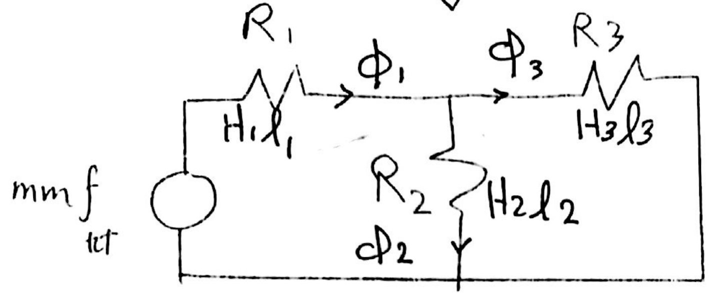
For Parallel Magnetic Circuit:





نفس قواعد الدوائر الكهربائية .

$$\phi_1 = \phi_2 + \phi_3$$

$$NI - H_1 l_1 = H_2 l_2 = H_3 l_3$$

$$NI - \phi_1 R_1 = \phi_2 R_2 = \phi_3 R_3$$

$$\therefore \; NI = \phi_1 \left(R_1 + \frac{R_2 * R_3}{R_2 + R_3} \right)$$

Stored energy in magnetic Circuit:

$$W_m = \frac{1}{2} \int_{vol} B \cdot H \, dv = \frac{1}{2} L I^2$$

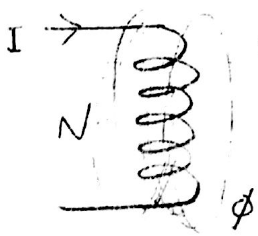
$$= \frac{1}{2} \mu \int_{vol} |H|^2 \, dv = \frac{1}{2\mu} \int_{volu} |B|^2 \, dv$$

Self Inductance: $L = \dfrac{\Lambda}{I}$ $\Lambda \sim$ Flux linkage

الفيض المترابط

$$= \frac{N\phi}{I} = N \int B \cdot ds$$

التيار : مصدر الفيض



$$\therefore \quad H_1 = H_{1n} + H_{1t}$$

$$\therefore \quad H_{1t} = H_1 - H_{1n} = (-2, 6, 4) - (-4, 4, 0)$$

$$= 2\bar{a}_x + 2\bar{a}_y + 4\bar{a}_z$$

Using Boundary Conditions

$$\therefore \quad H_{2t} = H_{1t} = 2\bar{a}_x + 2\bar{a}_y + 4\bar{a}_z$$

$$B_{2n} = B_{1n}$$

$$\therefore \quad \mu_2 H_{2n} = \mu_1 H_{1n}$$

$$\therefore \quad H_{2n} = \frac{\mu_1}{\mu_2}(H_{1n}) = \frac{5}{2}(-4\bar{a}_x + 4\bar{a}_y)$$

$$H_{2n} = -10\bar{a}_x + 10\bar{a}_y$$

$$\therefore \quad \bar{H}_2 = H_{2t} + H_{2n}$$

$$= -8\bar{a}_x + 12\bar{a}_y + 4\bar{a}_z \quad A/m$$

$$\therefore \quad B_2 = \mu_2 H_2 = \mu_{2r}\mu_0 H_2$$

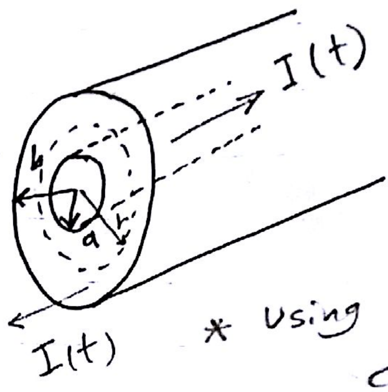
$$= -20.11\,\bar{a}_x + 30.16\,\bar{a}_y + 10.05\,\bar{a}_z \ \text{wb}/m^2$$

Called. Self Inductance

EX: $\boxed{2}$ Find the inductance of a Coaxial Cable Per unit length.

Solution

Using Ampere law



$$I_{enc} = \oint H \cdot dl$$

- for $a < \rho < b$

* Using cylindrical coordinates (ρ, ϕ, Z)

$$\oint H \cdot dl = I$$

$$H(2\pi\rho) = I$$

$$\therefore H = \frac{I}{2\pi\rho} a_\phi$$

$$\lambda = \cancel{N}\phi = \int_a^b B \cdot ds = \mu_0 \int_b^a H_\phi \cdot ds_\phi$$

$$= \mu_0 \frac{I}{2\pi} \int_{\rho=a}^{b} \int_{Z=0}^{1} \frac{1}{r} \, d\rho \, dZ \qquad d\rho\, dZ$$

$$= \frac{\mu_0 I}{2\pi} Z\Big|_0^1 \ln\rho\Big|_a^b$$

$$= \frac{\mu_0 I}{2\pi} \ln\frac{b}{a}$$

$$L = \frac{\lambda}{I} = \frac{\mu_0}{2\pi} \ln\frac{b}{a} \quad H/m$$

Ampere law $\leftarrow H \leftarrow B \leftarrow \phi \leftarrow \lambda$?

Note: $\mu = \frac{B}{H}$

EX [3] given:

Uniform Cross section Area $= 2\ Cm^2$

$N = 500$

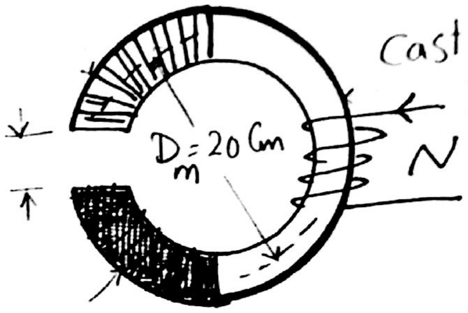
$\phi = 0.2\ mWb$

air-gap $\mu_{sheet} = 0.00333 = \frac{1}{300}$

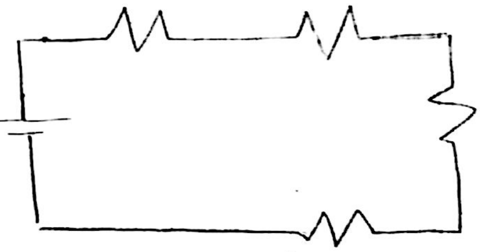
$\mu_{cast} = 0.001 = \frac{1}{1000}$

$\mu_{silicon} = \frac{1}{2300}$

Find I ?



⇓



$$B = \phi / A$$
$$= \frac{0.2 * 10^{-3}}{2 * 10^{-4}} = 1\ \text{Tesla}$$

$$R_g = \frac{l_g}{\mu_o A} = \frac{0.05}{\mu_o * 2 * 10^{-4}} = 198.94 * 10^6$$

$$R_{sheet} = \frac{l_{sh}}{\mu_{sh} A}, \quad l_{sh} = \frac{\pi D_m}{4} = \frac{\pi}{20}\ m$$

ربع محيط الدائرة

$$= \frac{\pi/20}{\frac{1}{300} * 2 * 10^{-4}} = 235.62 * 10^3\ AT/wb$$

$$R_{cast} = \frac{l_{cast}}{\mu_{cast} A}, \quad l_{cast} = \frac{\pi D_m}{2} = \frac{\pi}{10}$$

نصف محيط الدائرة

$$= \frac{\pi/10}{\frac{1}{1000} * 2 * 10^{-4}} = 1.57 * 10^6\ AT/wb$$

$$R_{silicon} = \frac{l_{silic}}{\mu_{silic} * A} \quad , \quad l_{silicon} = \frac{\pi D_m}{4} = \frac{\pi}{20} \text{ m}$$

$$= \frac{\pi/20}{\frac{1}{2300} * 2 * 10^{-4}} = 1.806 * 10^{6} \text{ AT/wb}$$

$$\therefore R_{tot} = R_{cast} + R_{sheet} + R_{sheet} + R_g$$
$$= 202.55 * 10^{6} \text{ AT/wb}$$

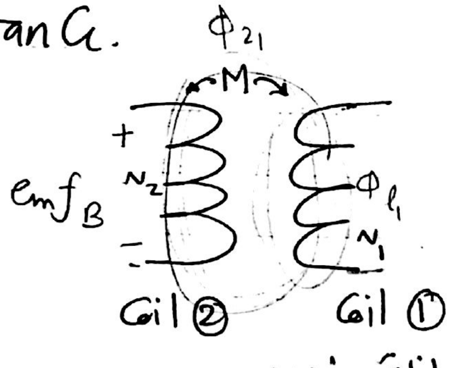
$$N I = \phi R_{tot}$$

$$500 \; I = 0.2 * 10^{-3} * 202.55 * 10^{6}$$

$$\therefore I = 81.022 \text{ A}$$

$$\underset{\text{Self-Induct}}{L} = \frac{NI}{\phi} = \frac{500 * 81.022}{0.2 * 10^{-3}} \text{ H}$$
$$= R$$

* **Mutual Inductance.**



$$M_{21} = \frac{\lambda_{21}}{I_1}$$

$$= \frac{N_2 \phi_{21}}{I_1}$$

ϕ_{21}: الفيض الناتج من الملف ① ومصدره التيار I_1 ويقطع الملف ②

Similarly

$$M_{12} = \frac{N_1 \phi_{12}}{I_2}$$

For linear magnetic Circuit:

$$M_{12} = M_{21} = M$$

Ex (4): Two Coils A & B

$\phi_{BA} = 60\%\ \phi_A$, $I_A = 6A$, $\phi_A = 1.8\ mWb$

, $N_B = 1500$ Turns

Find a) Mutual induct

b) $E_B \equiv$ emf induced in Coil B when I_A is reduced to Zero in 1 m Sec $\equiv \frac{\Delta I_A}{\Delta t} = \frac{6}{1 \times 10^{-3}}$

Solution

$$M_{BA} = \frac{N_B\, \phi_{BA}}{I_A}$$

$$= \frac{1500 \times 1.08 \times 10^{-3}}{6} = 0.27\ H$$

$$emf_B = M \frac{di_A}{dt} = M \frac{\Delta I_A}{\Delta t}$$

$$= 0.27 \times \frac{6}{1 \times 10^{-3}} = 1620\ \text{Volt}$$

* القوة الدافعة الكهربية المستحثة على الملف (B) نتيجة وجود فيض متغير مصدره الملف (A) ⇐ تنشأ عندما يكون هناك معدل تغير في الفيض (أي التيار) بالنسبة للزمن

$$\because \frac{di}{dt} \Rightarrow \frac{d\phi}{dt} \xrightarrow[\therefore]{\text{تنشأ}} emf$$

Force and Torque in magnetic Field

$$\bar{F} = Q\,\bar{u} \times \bar{B}$$

$\bar{B}$ → المجال المغناطيسي

$\bar{u}$ → السرعه الخطيه للشحنه

if there's additional Electricfield $\bar{E}$

$$\therefore \bar{F}_{TOT} = Q(\bar{E} + \bar{u} \times \bar{B})$$

Ex: given: $\bar{B} = 5\ mT\ (-\bar{a}_z)$

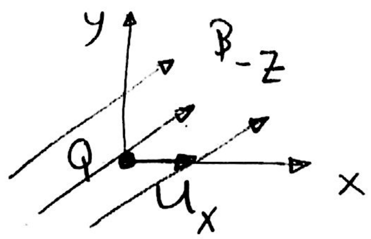
$u = 83.5\ Km/s\ (a_x)$

$Q = 1.6 \times 10^{-9}\ C$

what electric Field (E) present if $F_{net} = Zero$

Solution

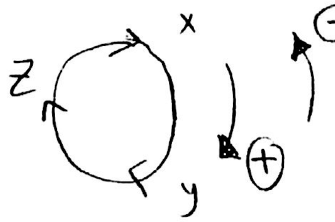
$$\bar{F}_{mag} = Q\,\bar{U} \times \bar{B}$$



$$= Q\ u(a_x) \times B(-a_z)$$

$$\bar{F}_{mag} = QUB\ \bar{a}_y \quad , \quad F_{elect} = Q\bar{E}$$

for $F_{net} = 0$



$$\therefore \bar{F}_{elect} = |F_{mag}|\ -\bar{a}_y$$

$$\therefore \cancel{Q}E = \cancel{Q}UB$$

$$= (83.5 \times 10^3) \times (5 \times 10^{-3})\ V/m$$

Magnetic Force on a differential Current element

$$\overline{dF} = dQ\,(\bar{u} \times \bar{B}) \quad , \quad I = \frac{dQ}{dt} \quad , \quad \frac{\bar{u}}{dt} = dl$$

$$\overline{dF} = I\,(\overline{dl} \times \bar{B})$$

$$\therefore \ \bar{F} = I(\bar{L} \times \bar{B}) \rightarrow \text{motor theory.}$$

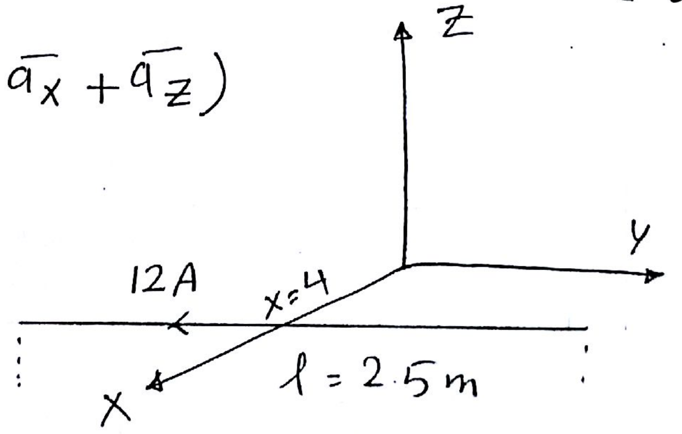
اتجاهه هو اتجاه التيار

Ex $\boxed{2}$: given: A Conductor length $L = 2.5\,m$ at $Z=0$, $X = 4\,m$, $I = 1.2\,A$ in $(-a_y)$

$$F = \frac{1.2 \times 10^{-2}}{\sqrt{2}}\,(-\bar{a}_x + \bar{a}_z)$$

Find B ?

$$F = I(\bar{L} \times \bar{B})$$



$$I(\bar{L} \times \bar{B}) = 12 \begin{vmatrix} a_x & a_y & a_z \\ 0 & -2.5 & 0 \\ B_x & B_y & B_z \end{vmatrix} = (-30\,B_z)\,a_x + (30\,B_x)\,a_z$$

$$= \frac{1.2 \times 10^{-2}}{\sqrt{2}}\,(-a_x + a_z)$$

$$\therefore B_z = B_x = \frac{4 \times 10^{-4}}{\sqrt{2}}\ T$$

$B_y \rightarrow$ have any value.

Work and Power

$$W = \int_{int}^{fin} \bar{F}_a \cdot d\bar{l}$$

ternal work
→applied work

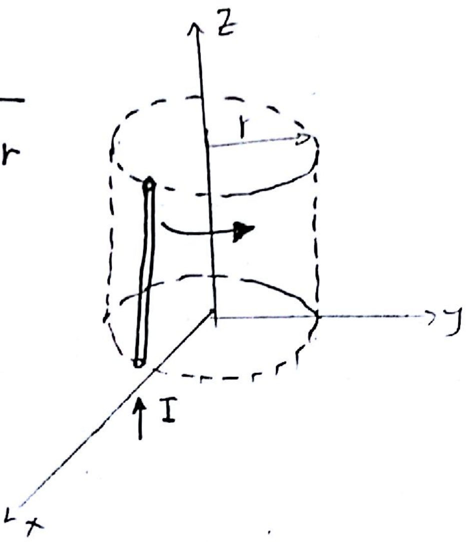
F_a: applied force from magnetic field, opposit to force

$$P = \frac{dW}{dt} = \frac{\bar{F}_a \cdot d\bar{l}}{dt}, \quad F_a = -I L \times B$$

$$P = \bar{F}_a \cdot \bar{u}$$

Ex ③ given: $B = B_0 \bar{a}_r$

Find F, W, P?



$$F_{mag} = I\bar{L} \times \bar{B}$$

$$= I\ L(\bar{a}_z) \times B_0(\bar{a}_r)$$

$$= I L B_0\ \bar{a}_\varphi$$

$$\bar{F}_a = -B_0 I L (\bar{a})_\varphi = B_0 I L (-\bar{a}_\varphi)$$

$$W = \int_{int}^{final} F_a \cdot dl \quad , \bar{dl} = r\, d\phi\ \bar{a}_\varphi$$

$$= -\int_0^{2\pi} B_0 I L r\ d\phi = -2\pi B_0 I L$$

full revolution

$$P = \frac{W}{t} = \frac{2\pi B_0 I L}{(60/\pi)}$$

Torque: $\bar{T} = \bar{r} \times \bar{F}$

$$= r(\bar{a}_r) \times B_0 I L(-\bar{a}_\varphi)$$